EXISTENCE AND CLASSIFICATION OF SINGULAR SOLUTIONS TO NONLINEAR ELLIPTIC EQUATIONS WITH A GRADIENT TERM

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ABSTRACT. Let Ω be a domain in $\mathbb{R}^N$ with $N \geq 2$ and $0 \in \Omega$. For $0 < m < 2$, $q > 0$ and $m + q > 1$, we obtain a complete classification of the behaviour near 0 (as well at ∞ if $\Omega = \mathbb{R}^N$) for all positive $C^1(\Omega \setminus \{0\})$ solutions of the elliptic equation $\Delta u = u^q |\nabla u|^m$ in $\Omega \setminus \{0\}$, together with corresponding existence results. We prove that (a) when $\Omega = \mathbb{R}^N$, any positive solution with a removable singularity at 0 must be constant; (b) If $q_* := \frac{N-m\,(N-1)}{N-2}$ for $N \geq 2$ and E denotes the fundamental solution of the Laplacian, then for $0 \leq q < q_*$, any positive solution has either a removable singularity at 0, or $\lim_{|x|\to 0} u(x)/E(x) \in (0, \infty)$ or $\lim_{|x|\to 0} |x|^{\vartheta} u(x) = \lambda$ with ϑ and λ uniquely determined positive constants. When $\Omega = \mathbb{R}^N$, we establish that any positive solution is radially symmetric and non-increasing with (possibly any) non-negative limit at ∞. (c) If, in turn, $q \geq q_*$ for $N \geq 3$, then 0 is a removable singularity for all positive solutions.

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