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Propagation of one and two-dimensional discrete waves under finite difference approximation

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We analyze the propagation properties of the numerical versions of one and two-dimensional wave equations, semi-discretized in space by finite difference schemes. We focus on high-frequency solutions whose propagation can be described, both at the continuous and semi-discrete level, by microlocal tools. We do it both for uniform and non-uniform numerical grids and also for constant coefficients and variable ones. The energy of continuous and semi-discrete high-frequency solutions propagates along bi-characteristic rays, but their dynamics differ from the continuous to the semi-discrete setting, because of the different nature of the corresponding Hamiltonians. One of the main objectives of this talk is to illustrate through accurate numerical simulations that, in agreement with the micro-local theory, numerical high frequency solutions can bend in an unexpected manner, as a result of the accumulation of the local effects introduced by the heterogeneity of the numerical grid. These effects are enhanced in the multi-dimensional case where the interaction and combination of such behaviors in the various space directions may produce, for instance, the rodeo effect, i.e. waves that are trapped by the numerical grid in closed loops, without ever getting to the exterior boundary. Our analysis allows explaining all such pathological behaviors. Moreover, our discussion also contributes to the existing theory about the necessity of filtering high-frequency numerical components when dealing with control and inversion problems for waves, which is based very much in the theory of rays and, in particular, on the fact that they can be observed when reaching the exterior boundary of the domain, a key property that can be lost through numerical discretization.

Joint works with Aurora Marica (Politehnica University of Bucharest) and Enrique Zuazua (Universidad de Deusto, Universidad Autónoma de Madrid and Sorbonne Universités, Paris)