

UNIVERSITY OF CRAIOVA
Faculty of Sciences
Department of Mathematics
Fundamental domain:
MATHEMATICS
Master Applied Mathematics

YEAR I

SUBJECT OF STUDY: Topics in Category Theory

NUMBER OF CREDITS: 8

SEMESTER: semester 1

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: To enable the students with the language of theory of categories, as well to present more mathematical results in the specific language of theory of categories.

CONTENT: A. Notion of category. Examples. Subcategory. Dual category. Duality principle. Product of categories.

B. Special morphisms in a category. Kernel (cokernel) for a couple of morphisms.

C. Functors. Examples. Remarkable functors. Functorial morphisms. Equivalent categories.

D. Representable functors; adjoint functors.

E. Products (coproducts) of objects in a category.

F. Inductive (projective) limits of objects in a category.

G. Fibred product; fibred coproduct.

H. Injective (projective) objects.

I. Applications of theory of categories in mathematical analysis and topology.

TEACHING LANGUAGE: Romanian

EVALUATION: written exam

BIBLIOGRAPHY:

1. M. Barry: *Theory of categories*, Academic Press, 1965.
2. D. Buşneag: *Categories of algebraic logic*, Ed. Academiei, Bucuresti, 2006.
3. P.J. Cameron: *Sets, Logic, and Categories*, Springer Undergraduate Mathematics Series, 1999.
4. S.M. Lane: *Categories for the Working Mathematician*, Springer, 1997.
5. C. Nastasescu: *Inele. Module. Categorii*, Ed. Academiei, Bucuresti, 1976.

SUBJECT OF STUDY: Special topics in functional analysis

NUMBER OF CREDITS: 6

SEMESTER: semester 1

TYPE OF COURSE: Fundamental, compulsory

OBJECTIVES: Main results of functional analysis and some applications from PDE's and convex functions theory are presented.

CONTENT:

1. Banach spaces and linear operators. Linear and continuous operators, the norm of an operator, equivalent norms. Uniform boundedness principle (Banach-Steinhaus).
2. The open map theorem, the closed graph theorem. The inverse of a linear and bounded operator.
3. The spectrum and the resolvent of an operator. Compact operators. Integral operators.
4. Hilbert spaces and orthogonal sums. Bases on Hilbert spaces. Orthogonal projections, the adjoint of an operator.
5. The diagonalization of a self-adjoint and compact operator. The spectrum of the self-adjoint and compact operators.
6. Hilbert Schmidt's theorem of spectral decomposition. Applications to Sturm-Liouville's problems.
7. Hahn-Banach's theorems. Consequences. Convex sets.
8. Separation theorems, Krein-Milman's theorem. Local convex spaces.
9. Weaker topologies. Alaoglu-Bourbaki's theorem. Reflexive spaces. The case of the L_p spaces, $p > 1$.
10. Nonlinear functional analysis theory. Differential calculus on Banach spaces. Elements of convex analysis on Banach spaces. The minimum of the convex functionals.
11. The fixed point theorem of Brouwer, Schauder and Schaeffer. Applications to PDE's.
12. Applications of functional analysis to PDE's. The convolution product, Dirac sequences, Weierstrass's approximation theorems.
13. The Fourier transforms. Schwartz's space, The Cauchy problem for the diffusion equation.
14. Dirichlet and Neumann problems.

TEACHING LANGUAGE: Romanian

EVALUATION: written examination

BIBLIOGRAPHY:

1. H. W. Alt, *Lineare Funktionalanalysis*, Springer-Lehrbuch, Berlin, 1992.
2. H. Brezis, *Functional analysis, Sobolev spaces and partial differential equations*, Springer, 2011.
3. C. P. Niculescu, *Probleme speciale de analiză funcțională*, Ed. Universitaria, Craiova, 2005.
4. W. Rudin, *Analyse fonctionnelle*, Ed. Ediscience International, 1995.
5. K. Yosida, *Functional Analysis*, 5th ed., Springer-Verlag, Berlin, 1995.
6. M. Willem, *Analyse fonctionnelle élémentaire*, Ed. Cassini, Paris, 2003.

SUBJECT OF STUDY: Applied nonlinear analysis**NUMBER OF CREDITS: 7****SEMESTER:** semester 1**TYPE OF COURSE:** Fundamental, Compulsory

OBJECTIVES: We develop some of the main techniques and principles in the modern nonlinear analysis, at the interplay with mathematical physics and numerical analysis. The course is conceived as a fundamental instrument and guide to do research in modern nonlinear analysis.

CONTENT:**A.** Logistic equations

Examples. The Keller-Osserman condition. Blow-up boundary solutions. Existence and uniqueness theorems for large solutions. The asymptotic behavior of the explosive solution. The Conjecture of H. Brezis.

B. Lane-Emden-Fowler singular type equations
Elliptic equations with singular term. The Crandall-Rabinowitz-Tartar theorem. Case of convection terms. Similarities with the logistic equation.

C. Nonlinear eigenvalue problems for nonhomogeneous differential operators
Elements of spectral theory for nonlinear operators. Orlicz-Sobolev spaces.
Nonhomogeneous differential operators.
Concentration of the spectrum. Open problems.

TEACHING LANGUAGE: Romanian**EVALUATION:** written examination**BIBLIOGRAPHY:**

1. H. Brezis, *Analiza functională: teorie, metode și aplicații*, Ed. Academiei, București, 2002 (translation from French by V. Radulescu).
2. M. Ghergu, V. Radulescu, *Singular Elliptic Problems. Bifurcation and Asymptotic Analysis*, Oxford Univ. Press, 2008.
3. V. Radulescu, *Singular phenomena in nonlinear elliptic problems*. In *Handbook of Differential Equations: Stationary PDEs*, vol. 4 (M. Chipot, Ed.), North-Holland, Amsterdam, 2007, pp. 483-591.
4. V. Radulescu, *Qualitative Analysis of Nonlinear Elliptic Partial Differential Equations*, Contemporary Mathematics and Its Applications, Hindawi, 2008

SUBJECT OF STUDY: Dynamical systems**NUMBER OF CREDITS: 7****SEMESTER:** semester 1**TYPE OF COURSE:** Fundamental, Compulsory

OBJECTIVES: Presentation of fundamental results regarding the dynamical systems associated to ordinary differential equations.

CONTENT: C1 : The phase plane: orbits, equilibria, separatrices.

C2 : The classification of plane linear systems. Topological conjugation, topological equivalence.

C3 : The Liapunov-Poincaré-Perron theorem on hyperbolic equilibria.

C4 : Stable and unstable manifolds of a hyperbolic equilibrium. The Hartman-Grobman theorem.

C5 : The Morse lemma: transforming functions into quadratic forms. Homogeneous differential systems : the Euler theorem, polar coordinates, desingularisation.

C6 : Central manifolds of a non-hyperbolic equilibrium. System reduction in stability analysis.

C7 : Lyapunov stability: first approximation, Lyapunov functions, Hamiltonian systems.

C8 : Limit-sets: α , ω . Homoclinic, heteroclinic orbits. Limit-cycles.

C9 : The Poincaré map. The Bendixson-Poincaré theory.

C10 : Periodic solutions and their center-stable manifolds.

C11 : Plane vector fields. Their index..

C12 : Nonautonomous dynamical systems: applied to shooting problems (steady solutions for a semilinear heat equation, nonlinear Dirac field equations).

C13 : Nonautonomous dynamical systems: applied to fluid mechanics (vortices in bounded regions).

C14 : The graphic displaying of orbits: software systems.

TEACHING LANGUAGE: Romanian

EVALUATION: colloquium

BIBLIOGRAPHY:

1. R.C. Robinson, An introduction to dynamical systems: continuous and discrete, Prentice Hall, Upper Saddle River, 2004
2. C. Chicone, Ordinary differential equations with applications, Springer-Verlag, New York, 2006

SUBJECT OF STUDY: Numerical Analysis for Partial Differential Equations

NUMBER OF CREDITS: 6

SEMESTER: semester 2

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: We introduce the fundamental methods of approximation of PDE and we analyse their stability and consistency

CONTENT:

1. **Introduction:** Types of PDE. Elliptic equation. Boundary conditions. Heat equation, wave equation, convection-diffusion equation.
2. **Fundamental notions:** Convergence. Consistency. Stability. Lax's theorem.
3. **Finite difference schema for equations of evolution:** Parabolic equation. Explicit and implicit methods. Hyperbolic equation. Ecuații parabolice. Explicit and implicit methods. Stability and convergence. Applications.
4. **Finite element method:** Lagrange finite element. Mesh of a regular domain. Families of triangulations. Approximation of the solutions of elliptic equation.
5. **Convergence of the finite element method:** Cea's Lemma. Conditions of convergence. Interpolation Lagrange in Sobolev spaces. Error evaluation in Lagrange interpolation. Error evaluations for finite element method.

6. Parabolic equation: Semi-discretization and total-discretization. Trapezoidal method. Stability. Convergence.

7. Hyperbolic equation: Semi-discretization and total-discretization. Newmark method. Stability. Convergence.

TEACHING LANGUAGE: Romanian

EVALUATION: written examination

BIBLIOGRAPHY:

1. H. Brezis: *Analyse fonctionnelle: Théorie et applications*, Masson, Paris, 1983;
2. P.G. Ciarlet: *Introduction à l'analyse numérique matricielle et à l'optimisation*, Masson, Paris, 1988;
3. P.G. Ciarlet: *The finite element method for elliptic problems*, North-Holland, Amsterdam, 1978;
4. K. Eriksson, D. Estep, P. Hansbo și C. Johnson: *Computational differential Equations*, Studentlitteratur, Lund, 1996;
5. P.A. Raviart and J.M. Thomas: *Introduction à l'analyse numérique des équations aux dérivées partielles*, Masson, Paris, 1983;
6. P. Rabier and J.M. Thomas: *Exercices d'analyse numérique des équations aux dérivées partielles*, Masson, Paris, 1985.
7. J. Strickwerda: *Finite difference schemes and partial differential equations*, Pacific Grove, California, 1989.

SUBJECT OF STUDY: Mathematical modeling with differential equations

NUMBER OF CREDITS: 8

SEMESTER: semester 1

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: To familiarize the students with models in mechanics, biology or economy, described by differential equations. To provide the methods in order to analyze such kind of models.

CONTENT: A. Modeling by ordinary differential equations

C₁. Modeling the motion of the material point.

C₂. Modeling the mass-spring system oscillations

C₃. Modeling the mass-spring-dashpot system oscillations

C₄. Modelling population dynamics

B. Modeling by partial differential equations

C₅. Elementes of mechanics of continua: the geometry of the deformation, displacement vector, infinitezimal stress tensor

C₆. General principles

C₇. The Cauchy Equations. Boundary conditions

C₈. Constitutive laws

C₉. The 3D- model in elastostatics

C₁₀. The antiplane model.

C₁₁. The plane model.

C₁₂. Classical problems in elastostatics

C₁₃. Modeling the contact between a deformable solid and a rigid obstacle

C₁₄. Modeling the contact between a deformable solid and a deformable foundation

TEACHING LANGUAGE: Romanian

EVALUATION: exam

BIBLIOGRAPHY:

- 1) J. Albery, C. Carstensen, S. A. Funken, *Remarks around 50 lines of Matlab: short finite element implementation*, Numerical Algorithms, 20 (1999), 117-137.
- 2) J. Albery, C. Carstensen, S. A. Funken, R. Klose, *Matlab implementation of the finite element method in elasticity*, 2002.
- 3) V. Barbu, *Ecuatii diferentiale*, Editura Junimea, 1985.
- 4) T.J. Hughes, *The Finite Element Method*, Prentice-Hall, 1987.
- 5) M. Shillor, M. Sofonea and J. Telega, *Models and Variational Analysis of Quasistatic Contact*, vol. 655, Springer, Berlin Heidelberg, 2004.
- 6) M. Sofonea and A. Matei, *Variational Inequalities with Applications. A study of Antiplane Frictional Contact Problems*, *Advances in Mechanics and Mathematics*, Vol. 18, Springer, 2009.
- 7) M. Sofonea and A. Matei, *Mathematical Models in Contact Mechanics*, London Mathematical Society, Lecture Note Series 398, 280 pages, Cambridge University Press, 2012.
- 8) A. Quarteroni, F. Saleri, P. Gervasio, *Calcul Scientifique, Cours, exercices corrigés et illustrations en Matlab et Octave*, Springer, 2010.

SUBJECT OF STUDY: Equations of evolutions

NUMBER OF CREDITS: 6

SEMESTER: semester 2

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: We study the theory of existence,

uniqueness, regularity and asymptotic behavior of solutions of evolution equations, linear and semi-linear.

CONTENT:

1. Introduction
2. Unbounded operators. Semigroups. Hille-Yosida theorem. Classical and weak solutions of linear homogeneous and nonhomogeneous equations of evolution.
3. Diagonalized and sectorial operators in Hilbert spaces. Scales of functional spaces. Fundamental properties.
4. Semi-linear heat equation: local and global existence. Explosion in finite time.
5. Semi-linear wave equation: local and global existence. Explosion in finite time.
6. The study of the asymptotic behavior of solutions. LaSalle invariance principle.

TEACHING LANGUAGE: Romanian

EVALUATION: written examination

BIBLIOGRAPHY:

1. H. Brezis: *Analyse fonctionnelle: Théorie et applications*, Masson, Paris, 1983.
2. V. Barbu: *Probleme la limită pentru ecuațiile cu derivate parțiale*, Ed. Academiei, București, 1993.
3. T. Cazenave and A. Haraux: *Introduction aux problèmes d'évolution semi-lineaires*, Ellipses, Paris, 1990.

SUBJECT OF STUDY: Ordered algebraic structures

NUMBER OF CREDITS: 6

SEMESTER: semester 2

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: To enable the students with the fundamental results from theory of ordered groups and rings, the category of ordered sets and the category of distributive bounded lattices and with the some categories of algebras of classical logic (Boole, Heyting, Stone, Hilbert) and fuzzy (residuated lattices, MV-algebras, BL-algebras).

CONTENT:

- A. The category of ordered sets;
- B. The category of bounded distributive lattices;
- C. The category of ordered groups and rings;

D.The category of some ordered algebras of classical logic (Boole, Stone, Heyting, Hilbert, etc);

E.The category of some ordered algebras of fuzzy logic (Residuated lattices, MV-algebras, BL-algebras);

TEACHING LANGUAGE: Romanian

EVALUATION: written examination

BIBLIOGRAPHY:

1. Balbes,R., Dwinger,Ph. : *Distributive lattices*, University of Missouri Press, 1974.
2. Bigard, A., Keimel, K., Wolfenstein, S. : *Groupes et anneaux reticules*, Lectures Notes in Mathematics, 608, Springer, Berlin, 1971.
3. Blyth, T.S. : *Lattices and Ordered Algebraic Structures*, Springer-Verlag London Limited, 2005.
4. Buşneag, D.: *Categories of algebraic logic*, Ed. Academiei, Bucuresti, 2006.
- 5.Cignoli, R., D'Ottaviano, I, Mundici, D. *Algebraic foundation of many-valued Reasoning*, Kluwer Academic Publishers, 2000.
- 6.Nastasescu, C. : *Teoria dimensiunii in algebra necomutativa*, Ed. Academiei, 1983.
- 7.Turunen, E.: *Mathematics Behind Fuzzy Logic*, Physica-Verlag, 1999.

SUBJECT OF STUDY: Research methodology

NUMBER OF CREDITS: 6

SEMESTER: semester 1

TYPE OF COURSE: complementary discipline, compulsory

OBJECTIVES: To familiarize the students with the main tools of the scientific research.

CONTENT: C₁. Typing math into Latex.

C₂. Professional english.

C₃. Current research directions in Applied Mathematics

C₄. Research problems through examples

C₅. Prestigious journals. Prestigious publishers.

C₆. Data base for mathematicians. Documentation

C₇. A research topic

C₈. Making a plan and writing a paper

C₉. Cover letter

C₁₀. Preparing a Manuscript for Submission to a journal.

C₁₁. Galley proofs.

C₁₂. Dissemination of research results (scientific seminar, poster, conference, publications in journals)

C₁₃. The review.

C₁₄. Research project. Research report.

TEACHING LANGUAGE: Romanian

EVALUATION: Colloquium (C)

BIBLIOGRAPHY:

1. Mihaela St. Radulescu, Metodologia Cercetarii Stiintifice, Editura Didactica si Pedagogica, 2006.
2. Ethical guidelines of the American Mathematical Society,
www.ams.org/secretary/ethics.html
- 3.Mathematics Subject Classification
www.ams.org/msc/
- 4.The Mathematical Association of America,
www.maa.org
- 5.Albert Einstein, Cum vad eu lumea. Teoria relativitatii pe intelesul tuturor (Principiile cercetarii) Editia a II-a, Humanitas, Bucuresti, 2000.

Year II

SUBJECT OF STUDY: Spectral theory of differential operators

NUMBER OF CREDITS: 8

SEMESTER: semester 3

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: The main goal of the course is to introduce students to the spectral theory of differential operators, starting with the case of linear operators, continuing with the case of nonlinear operators and concluding with presenting some results regarding the spectrum of nonlinear and nonhomogeneous operators.

CONTENT:

1.The spectrum of the Laplace operator:

Description. The isolation and simplicity of the first eigenvalue. The variational characterization of the first eigenvalue.

2.The spectrum of the p-Laplace operator:

Description. The isolation and simplicity of the first eigenvalue. The variational characterization of the first eigenvalue.

3.The spectrum of weighted Laplace and p-Laplace operators.

4. Spectral problems related to the $p(x)$ -

Laplace operators: Definition of the variable exponent Lebesgue and Sobolev spaces. Basic properties of variable exponent Lebesgue and Sobolev spaces. The study of some eigenvalue problems involving $p(x)$ -Laplace type operators. Comparison with the case when $p(x)=\text{constant}$.

5. Spectral problems related to nonhomogeneous operators in Orlicz-Sobolev spaces: Definition of Orlicz and Orlicz-Sobolev spaces. Basic properties of Orlicz spaces. The study of some eigenvalue problems involving nonhomogeneous operators in Orlicz-Sobolev spaces.

TEACHING LANGUAGE: Romanian

EVALUATION: Colloquim

BIBLIOGRAPHY:

1. W. Allegretto, *Principal eigenvalues for indefinite-weight elliptic problems in $\mathbb{R}^N$* , Proceedings of the American Mathematical Society **116**, Vol. 3, 1992, 701-706.
2. A. Anane, *Simplicité et isolation de la première valeur propre du p -laplacien avec poids*, C. R. Acad. Sci. Paris, **305**, Série I, 1987, 725-728.
3. H. Brezis, *Analyse fonctionnelle. Théorie et applications*, Masson, Paris, 1992.
4. E. B. Davies, *Spectral Theory and Differential Operators*, Cambridge University Press, 1995.
5. L. Diening, P. Harjulehto, P. Hasto and M. Ruzicka, *Lebesgue and Sobolev spaces with variable exponents*, Lecture Notes in Mathematics, Vol. 2017 Springer-Verlag, Berlin, 2011.
6. Y. Egorov si V. Kondratiev, *On Spectral Theory of Elliptic operators*, Birhauser, Berlin, 1996.
7. P. Lindqvist, *Notes on the p -Laplace equation*, Lectures at the Summer School in Jyväskylä in August 2005.
8. M. Mihailescu si V. Radulescu, *On a nonhomogeneous quasilinear eigenvalue problem in Sobolev spaces with variable exponent*, Proceedings Amer. Math. Soc. **135** (2007), 2929-2937.
9. A. Szulkin si M. Willem, *Eigenvalue problems with indefinite weight*, Studia Mathematica **135** (2) (1999), 191-201.
10. M. Willem, *Analyse harmonique réelle*, Birhauser, Basel, 1996.

SUBJECT OF STUDY: Ergodic theory of dynamical systems

NUMBER OF CREDITS: 8

SEMESTER: semester 3

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: Presentation of fundamental results in the ergodic theory of dynamical systems

CONTENT:

- C1:** Classical dynamical systems: measure spaces, uniparametric transformation groups.
C2: Geodesic flows and Hamiltonian systems. Invariant measures.
C3: Mean time and space theory: the Birkhoff-Khinchin theorem.
C4: Ergodicity and mixing on the unit-circle.
C5 : Spectral invariants (Koopman).
C6 : The Lebesgue spectrum, Bernoulli schemea.
C7: Entropy, partitions.
C8 : Anosov systems and structural stability I.
C9: Anosov systems and structural stability II.
C10: Stable systems: canonical mappings, fixed points and periodic motions.
C11: Integrable systems, action-angle variables.
C12: Averaging, transitions.
C13: Smale theory I.
C14: Smale theory II.

TEACHING LANGUAGE: Romanian

EVALUATION: written exam

BIBLIOGRAPHY:

1. V.I. Arnold, *Geometrical methods in the theory of ordinary differential equations*, Springer-Verlag, New York, 1988
2. V.I. Arnold, A. Avez, *Ergodic problems of classical mechanics*, W.A. Benjamin, Inc., New York, 1968

SUBJECT OF STUDY: Oscillation theory

NUMBER OF CREDITS: 7

SEMESTER: semester 3

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: Presentation of fundamental results regarding the oscillation of solutions to ordinary differential equations

CONTENT:

- C1 :** Comparison of differential equations: the Sturm-Picone theorem.
- C2 :** Integral oscillation criteria: theorems of Fite, Mikusinski. Analysis of derivatives : the lemmas of Kiguradze. The Staikos-Philos theorem for functional differential equations.
- C3 :** Local extrema, the Trench nonoscillation theorem, integrations of the adiabatic oscillator.
- C4 :** Nonlinear first-order equations: Hartman's theorems.
- C5 :** The linear nonoscillation theorem. The Putnam oscillation criterion. Linearizing equations (Erbe).
- C6 :** Kamenev-type criteria.
- C7 :** General weighted averages.
- C8 :** Asymptotic developments according to Poincaré-Perron.
- C9 :** The Hartman-Wintner theorem of asymptotic integration.
- C10 :** Nonlinear oscillations using Riccati expressions.
- C11 :** The theorem of Atkinson on the oscillation of the Emden-Fowler equations. The sublinear case (Heidel).
- C12 :** The Waltman nonoscillation criterion. The Wong-Coffman theorems.
- C13 :** The Hale-Onuchic theory of asymptotic integration.
- C14 :** Nonoscillatory equations: the Hille-Wintner theorem.

TEACHING LANGUAGE: Romanian**EVALUATION:** colloquim**BIBLIOGRAPHY:**

- 1.O.G. Mustafa, Oscilațiile ecuațiilor diferențiale ordinare, Editura Sitech, Craiova, 2007
- 2.O.G. Mustafa, Integrarea asimptotică a ecuațiilor diferențiale ordinare în cazul neautonom, Editura Sitech, Craiova, 2006

SUBJECT OF STUDY: Financial mathematics**NUMBER OF CREDITS:** 7**SEMESTER:** semester 3**TYPE OF COURSE:** Fundamental, Compulsory

OBJECTIVES: The aim of the course is to to equip our students with a solid foundation in mathematics and in doing so provide them with practical knowledge that they can successfully apply to complicated financial models. The course

develop derivative securities valuation, portfolio structuring, risk management and scenario simulation through the binomial pricing model and arbitrage pricing theory

CONTENT: C₁ : Financial markets and derivative securities. No-arbitrage condition. Call options. Put options. Forward contracts. Examples

C₂ : No arbitrage price of an option for the binomial model. Delta-hedging formula. Example

C₃: First Fundamental Theorem of Asset Pricing for a multiperiod , finite state model. Example

C₄: Computational considerations. Examples

C₅: Probability theory and discrete-time stochastic processes. Distributions. Random variables. Examples

C₆: Risk-neutral measure and option pricing . Expectation, information, and σ -algebras. Jensen's inequality . Exemple

C₇: Conditional expectations. Fundamental properties. Exemples

C₈: Martingales. The discounted stock price. The discounted wealth process

C₉: Risk-neutral pricing formula. Cash flow valuation

C₁₀: Markov Processes. Lemma of independence. Exemples: Stock price, Non – Markov process

C₁₁: Change of measure. Radon-Nikodým derivative process

C₁₂: State price density. Optimal investment.Examples

C₁₃: Derivatives securities. Non-path- dependent derivatives. Stopping times. The Optional Sampling Theorem

C₁₄: Random walk. First Passage Times. Reflection Principle. Example

TEACHING LANGUAGE: Romanian**EVALUATION:** colloquim**BIBLIOGRAPHY:**

- 1.K.Back, *A Course in Derivative Securities: Introduction to Theory and Computation*, Springer, 2005
- 2.M.Baxter and A. Rennie, *Financial Calculus: An Introduction to Option Pricing*, Cambridge, 1996
- 3.J.C.Hull, *Options, Futures, and other Derivatives*, 6th Edition, Prentice Hall, 2006

- 4.P.Hunt and J.Kennedy, *Financial Derivatives in Theory and Practice*, Wiley, 2004
 5.S.R.Pliska, *Introduction to Mathematical Finance: Discrete Time Models*, Blackwell, 1997
 6.S.E.Shreve, *Stochastic calculus and Finance I: Binomial Model*, Springer, 2004
 7.P.Wilmott, *Paul Wilmott on Quantitative Finance*, 2nd edition, Wiley, 2006

SUBJECT OF STUDY: Riemannian Geometry

NUMBER OF CREDITS: 8

SEMESTER: semester 4

TYPE OF COURSE: Fundamental, Compulsory

OBJECTIVES: We aim to generalize the knowledge concerning differential calculus in the specific framework of Riemannian manifolds. The students should comprehend the concept of curvature; the constant curvature canonical manifolds such as the sphere S^2 and the hyperbolic half-plane will be presented as examples of non-euclidean geometries. We generalize in the new setting notions like right lines, measure, integral, divergence operator, gradient operator, Laplace operator. We present the exponential map and offer examples in the case of Lie fundamental groups ($SO(3)$, $SU(2)$, $Spin(2)$) and their corresponding Lie algebras.

CONTENT:

A. Differentiable manifolds (2 lectures)

C₁: Introduction in the theory of differentiable manifolds

C₂: Lie groups as examples of differentiable manifolds.

B. Differential calculus on manifolds (3 lectures)

C₃: Tangent vectors on differentiable manifolds

C₄: Tangent vector fields. The flow generated by a tangent vector field.

C₅: Linear connection. Curvature tensor and torsion tensor.

C. Riemannian structures (3 lectures)

C₆: Riemannian manifolds.

C₇: Levi-Civita connection.

C₈: Canonical manifolds of constant curvature (S^n , R^n , H^n).

D. Theory of geodesics on manifolds (3 lectures)

C₉: Geodesic curves.

C₁₀: Jacobi fields. Exponential map.

C₁₁: Variational theory of geodesics. The first and second variation formula.

E. Differential operators on manifolds (3 lectures)

C₁₂: Integration on compact Riemannian manifolds.

C₁₃: Canonical differential operators on Riemannian manifolds.

C₁₄: Green formula on compact Riemannian manifolds.

TEACHING LANGUAGE: Romanian

EVALUATION: exam

BIBLIOGRAPHY:

1. M. Craioveanu, M. Puta and Th.M. Rassias, *Introducere în Geometria Diferențială*, Edit. Mirton, Timișoara, 2004.
2. L. Ornea, A. Turtoi, *O Introducere în Geometrie, ed. a II-a*, Ed. Theta 2011
3. Gh. Murarescu. *Geometrie diferențială. Curs*. Tipografia Universității din Craiova, 2004.
4. T. Sakai, *Riemannian Manifolds*, Trans. of Math. Monographs, vol 149, Am. Math. Soc., Rhode Island, 1996.
5. M. do Carmo, *Riemannian Geometry*, Birkhauser Verlag, 1992.
6. W. A. Poor, *Differential Geometric Structures*, McGraw-Hill, New York, 1981.
7. M. L. Curtis, *Matrix Groups*, Springer-Verlag, New York, 1984.

SUBJECT OF STUDY: Bifurcation Theory and Applications in Economy

NUMBER OF CREDITS: 8

SEMESTER: semester 4

TYPE OF COURSE: Specialized discipline /Compulsory

OBJECTIVES: We present of concepts and fundamental results of the bifurcation theory of continuous and discrete dynamical systems and their application to models from biology and economics.

CONTENT: 1.Topologic equivalence of dynamical systems. Topologic classification of equilibria and fixed points. Bifurcation and bifurcation diagrams. Structural stability. Normal forms. Center manifolds. Models from biology and economy: Van der Pol, Hodgkin-Huxley, prey-predator, business cycle models.

2.Codimension one bifurcation of equilibria in continuous-time dynamical systems: saddle-node bifurcation, Hopf bifurcation.

3. Codimension one bifurcation of fixed points in discrete-time dynamical systems: fold bifurcation, flip bifurcation, Neimark-Sacker bifurcation.

4.Bifurcations of homoclinic and heteroclinic orbits. Andronov-Leontovich theorem, Shil'nikov theorems, Melnikov integral.

5.Codimension two bifurcation of equilibria in continuous-time dynamical systems: cusp bifurcation, Bautin bifurcation, Bogdanov-Takens bifurcation, fold-Hopf bifurcation, Hopf-Hopf bifurcation.

6.Codimension two bifurcation of equilibria in discrete-time dynamical systems: cusp bifurcation, generalized flip bifurcation, bifurcations Neimark-Sacker bifurcation, resonance, fold-flip bifurcation. Numerical analysis of bifurcations. The XPP software

TEACHING LANGUAGE: Romanian

EVALUATION: colloquium

BIBLIOGRAPHY:

1. Arrowsmith D.K., Place C.M. *An introduction to dynamical systems*, Cambridge University Press, 1994.

2.Chow S.N., Li C., Wang D., *Normal forms and bifurcations of planar vector fields*, Cambridge Univ. Press, Cambridge, 1994.

3.Chow S.N., Hale J., *Methods of bifurcation theory*, Springer, New-York, 1982.

4.Ermentrout B. XPPAUT, <http://www.math.pitt.edu>

5.Ermentrout B. *Simulating, analyzing, and animating dynamical systems: a guide to XPPAUT for researchers and students*, SIAM, 2002.

6.Guckenheimer J., Holmes P. *Nonlinear oscillations, dynamical systems and bifurcations of vector fields*, Springer, New-York, 1983.

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11.Sterpu M., Rocşoreanu C. *Modeling and simulation of economic processes*, Editura Universitaria, Craiova, 2007.

12.Tu P. *Dynamical systems. An introduction with applications in economics and biology*, Springer, Berlin, 1994.

SUBJECT OF STUDY: Elements of cryptography

NUMBER OF CREDITS: 7

SEMESTER: semester 4

TYPE OF COURSE: specialized discipline, fundamental

OBJECTIVES: To have a knowledge of the basic notions and principles of cryptology; to know the major cryptosystems with secret key, DES and AES; to study the most popular public-key cryptosystems and their security; to apply mathematical notions in solving applicative problems.

CONTENT:

A. Basic notions (2 courses)

C₁ : Basic notions. Information security and cryptography. Basic concepts. Brief history of cryptography.

C₂ : Special classes of functions: One way functions, trapdoor, hash. Detecting error and methods of correction. Generating random numbers.

B. Symmetric-key encryption (4 courses)

C₃ : Symmetric-key encryption. Substitution cyphers: monoalphabetic (Cesar, affine), polialphabetic (Vigenere, Playfair, Hill). Cryptanalysis of such cyphers.

C₄ : Data Encryption Standard (DES). Product cyphers. Feistel cipher. Description of the cryptographic scheme DES. Utility of DES. Cryptosystems related with DES.

C₅ : Various attacks on DES. Meet in the middle, differential and linear cryptanalysis.

C₆ : Advanced Encryption Standard (AES). History. Description of the finalist cryptosystems for AES (Mars, RC6, Serpent, Twofish). AES.

C. Public-key cryptography (5 courses)

C₇ : Basic notions. The security of secret-key cryptosystems. Symmetric-key vs public-key cryptography.

C₈ : *RSA public-key cryptosystem*. Description. Implementation. *RSA* encryption in practice.

C₉ : *Security of RSA*. Relation to factoring. Small encryption exponent e . Message concealing. Another attacks.

C₁₀ : *El Gamal public-key cryptosystem*. Description. Security of discrete logarithms. Generalized El Gamal encryption.

C₁₁ : *Knapsack public-key encryption*. Merkle-Hellman encryption. Chor-Rivest public-key encryption.

D. Digital signature (2 courses)

C₁₂ : Introduction. Basic notions. A classification of digital signature schemes and short presentation. Types of attacks on signature scheme.

C₁₃ : *RSA* signature and possible attacks. El Gamal signature. Digital standard signature (DSS).

E. Data base security and secret sharing (1 course)

C₁₄ : Basic notions. Examples

TEACHING LANGUAGE: Romanian

EVALUATION: exam

BIBLIOGRAPHY:

1. Buşneag, D., Boboc, F., Piciu, D., *Aritmetică şi teoria numerelor*, Editura Universitaria, Craiova, 1999.

2. Dan, C., *Algoritmi în teoria numerelor*, Editura Universitaria, Craiova, 2005.

3. Koblitz, N., *A Course in Number Theory and Cryptography*, ed. a II-a, Springer-Verlag, Berlin, 1994.

4. Knut, D.E., *The Art of Computer Programming*, vol. I, ed. a II-a, Addison-Wesley, 1973.

5. Menezes, A., Oorschot, P., Vanstone, S., *Handbook of Applied Cryptography*, CRC Press, Boca Raton, Florida, 1998.

6. Yan, Song Y., *Number theory for computing*, ed. a II-a, Springer Verlag, 2002.

SUBJECT OF STUDY: The mathematics of contact models

NUMBER OF CREDITS: 7

SEMESTER: semester 4

TYPE OF COURSE: complementary discipline, obligatory

OBJECTIVES : To familiarize the students with modern techniques of variational calculus. To give variational formulations for a class of contact models. To introduce the notion of weak solution for each proposed mechanical model. To study the existence, uniqueness and stability of the weak solution.

CONTENT: A. Abstract variational problems

C₁. Variational equations.

C₂. Stationary variational inequalities.

C₃. Evolutionary variational inequalities.

C₄. Quasi-variational inequalities.

C₅. Stationary mixed variational problems.

C₆. Evolutionary mixed variational problems.

C₇. Systems of variational inequalities.

B. Boundary value problems in Contact Mechanics

C₈. Frictionless bilateral contact problems in elastostatics.

C₉. Frictionless unilateral contact problems in elastostatics.

C₁₀. Frictional bilateral contact problems in elastostatics.

C₁₁. Contact problem with prescribed normal stress and dry friction, in elastostatics.

C₁₂. Bilateral contact problems with slip dependent friction, in elastostatics.

C₁₃. Frictionless bilateral contact problems, in viscoelasticity.

C₁₄. Frictional bilateral contact problems, in viscoelasticity.

TEACHING LANGUAGE: Romanian

EVALUATION: written examination

BIBLIOGRAPHY:

1.I. Ekeland and R. Temam, *Convex analysis and variational problems* (Studies in mathematics and its applications; 1) North-Holland Publ. Comp. 1976.

2.J. Haslinger, I. Hlavacek and J. Necas, *Numerical methods for unilateral problems in solid mechanics*, in *Handbook of Numerical Analysis*, vol IV, North – Holland, Amsterdam, 1996, pp 313-485.

3.S. Hübner, A. Matei and B. I. Wohlmuth, *Efficient algorithms for problems with friction*, *SIAM Journal on Scientific Computing*, vol. 29, 70-92, 2007.

3.D. Kinderlehrer and G. Stampacchia, *An Introduction to Variational Inequalities and Their Applications*, SIAM, 2000.

- 4.A. Matei and R. Ciurcea, Contact problems for nonlinearly elastic materials: weak solvability involving dual Lagrange multipliers, ANZIAMJ, ISSN1445-8735, DOI:10.1017/S1446181111000629, vol. 52, 160-178, 2010.
- 5.A. Matei and R. Ciurcea, Weak solvability for a class of contact problems, Annals of the Academy of Romanian Scientists Series on Mathematics and its Applications Volume 2, Number 1, 2010, pp. 25-44, ISSN 2066-6594.
- 6.A. Matei, A variational approach via bipotentials for unilateral contact problems, Journal of Mathematical Analysis and Applications, <http://dx.doi.org/10.1016/j.jmaa.2012.07.065>.
- 7.M. Sofonea and A. Matei, Variational Inequalities with Applications. A study of Antiplane Frictional Contact Problems, Advances in Mechanics and Mathematics, Vol. 18, Springer, 2009.
- 8.M. Sofonea and A. Matei, Mathematical Models in Contact Mechanics, London Mathematical Society, Lecture Note Series 398, 280 pages, Cambridge University Press, 2012.