On the controllability of a one-dimensional fluid-structure model

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The aim of this poster is to present some recent results, obtained in [1], concerning the local exact controllability of the following nonlinear system :

$$\begin{cases} \dot{v}(t,y) - v_{yy}(t,y) + v(t,y)v_y(t,y) = 0 & t \in (0,T),\ y \in (-1,1),\ y \neq h(t) \\ \dot{h}(t) = v(t,h(t)) + u(t) & t \in (0,T) \\ m\ddot{h}(t) = [v_y](t,h(t)) & t \in (0,T) \\ v(0,y) = v_0(y) & y \in (-1,1) \\ h(0) = h_0, \qquad \dot{h}(0) = h_1. & \end{cases} \tag{1}$$

This system models the motion of a point swimmer in a one dimensional fluid. More precisely, v denotes the eulerian velocity of the fluid (governed by the viscous Burgers equation) filling the interval $(-1,1)$ whereas h indicates the position of the point swimmer. The system is controlled by the relative velocity of the point mass with respect to the fluid, denoted by $u(t)$.

The main result in [1] is given by the following theorem.

Theorem 1. *Let $T > 0$. There exists a strictly positive constant r such that for every $h_0 \in (-1,1)$, $h_1 \in \mathbb{R}$ and $v_0 \in H_0^1(-1,1)$ small enough in the sense*

$$|h_0| + |h_1| + \|v_0\|_{H_0^1(-1,1)} \leq r, \tag{2}$$

there exists a control $u \in C([0,T])$ such that the solution v of (1) satisfies

$$v \in L^2(0,T;H^2((-1,1)\ \{h(t)\})) \cap C([0,T],H_0^1(0,1)), \qquad h \in C^1([0,T]) \tag{3}$$

and

$$v(T) = 0, \qquad h(T) = 0, \qquad \dot{h}(T) = 0. \tag{4}$$

Leading ideas of the proof are presented in this poster. Moreover, some numerical simulations confirm the exact local controllability of the model.

References

[1] Cîndea, N., Micu, S., Rovenţa, I., and Tucsnak, M. (2013). Controllability of a one dimensional fluid-structure model. Preprint.

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