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Modelling, Analysis and LQ-Optimal Control of Boundary Control Systems with Boundary Observation

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Objectives: Boundary control problems with observation often feature unbounded operators along with the homogeneous dynamics. However, this characteristic should better be avoided in order to achieve an acceptable trade-off between the cost of modelling and the efficiency of methods of resolution of control problems. Our aim is to establish an extended differential system involving no unbounded operators except for the dynamics generator. We consider, under suitable conditions, a change of variables for the state and input, as well as an approximation of the output which is based on the resolvent operator. The main reason behind this choice is the well-posedness of the extended system and the conservation of the C_0-semigroup properties. Next, we report a method of resolution of the LQ-optimal control problem for such systems, which is based on the problem of spectral factorization and the resolution of a diophantine equation [1], which is an alternative to the resolution of the well known operator Riccati equation.

Summary: We consider an abstract boundary control system with observation, with unbounded boundary control and observation linear operators. With appropriate conditions and a change of variables, one can build an extended differential system where the control and observation operators are bounded. The main output component of this system is an approximation of the nominal output based on the resolvent operator, while the boundary input is now a state component and its variation rate is the new input. This extension is based on developments which can be found in [2]. It preserves many interesting properties of the nominal system, including (approximate) reachability, detectability or observability and several spectral properties, while keeping the state-space framework with bounded linear operators and the dynamics generating a C_0-semigroup [3].
Any LQ-optimal control problem for the nominal system can be rewritten in a nearly equivalent form for the extended system. Since the extended system is unstable, we first find a stabilizing feedback and a right-coprime fraction of the transfer function in order to compute the spectral density (or Popov function). Then an approximate invertible spectral factor is computed by using the method of symmetric pole-zero extraction, which consists in simultaneously extracting the zeros and poles from the spectral density in order to build a sequence of products converging towards the spectral factor [8]. In practical implementation, the product is truncated after a determined threshold. The construction of the elementary factors is heuristic and can be seen as an extension of the method described in [6] for rational spectral densities. Finally, we solve the diophantine equation by computing residues of the spectral factor and e.g. decomposing the optimal feedback coefficients in a biorthonormal basis of the state space (depending on the application). Once the design of the optimal state feedback is complete, it can be applied to the nominal system [4].
This methodology is applied to an example. We study a Sturm-Liouville operator modelling phenomena of diffusion, convection and reaction, resulting in dynamics based on PDEs and thus in an infinite-dimensional state-space system. This particular operator is used in the modelling of biochemical processes, such as nonisothermal tubular reactors with axial dispersion, biochemical reactors, UV disinfection and many others (see e.g. [5],[7]). Some of these systems feature bilinear or nonlinear components and must be linearized in order to be described by such dynamics.

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