Some Notes On the Structure of Robust Controllers for Distributed Parameter Systems

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One of the main results of classical control theory of finite-dimensional linear systems is the Internal Model Principle (IMP) due to Francis and Wonham [3], and Davison [1, 2]. This principle asserts that any error feedback controller which achieves closed loop stability also achieves robust output regulation if and only if the controller contains a suitably duplicated model of the dynamic structure of the exosystem which generates the reference and disturbance signals which the controller is required to track/reject.

The approach of Francis and Wonham is based on geometric theory. Davison's approach is non-geometric and leads to a remarkably simple result showing that a robust controller can be divided into two parts: a servocompensator and a stabilizing controller. The servocompensator contains an internal model of the dynamics of the the reference and disturbance signals in the form of a p-copy of the exosystem, where p is the dimension of the output space. The role of the stabilizing controller is to stabilize the extended system consisting of the servocompensator and the plant.

In this paper we use a new characterization of IMP based on the Internal Model Structure (IMS) of Immonen [4]. The IMS has been shown [5] to be equivalent to so called $\mathcal{G}$-conditions. Using the $\mathcal{G}$-conditions we show that if the reference and disturbance signals are generated by a finite-dimensional exosystem, then the controller can be decomposed into a servocompensator and a stabilizing controller generalizing Davison's result to infinite-dimensional plants. This also gives a new proof for the finite-dimensional case.

References

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