WELL-POSEDNESS AND CONTROLLABILITY ISSUES FOR SWITCHING MODELS IN CARDIOLOGY

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Abstract. In this poster, we will consider the Mitchell-Schaeffer model [6]

$$\left\{\begin{array}{ll} v_t = \triangle v + \frac{hv^2(1-v)}{\tau_{in}} - \frac{v}{\tau_{out}} + J_{stim}, & x\in\Omega, t\in(0,T] \\ h_t = \left\{\begin{array}{ll} -\frac{h}{\tau_{close}}, & v > v_{gate}, \\ \frac{1-h}{\tau_{open}}, & v < v_{gate} \end{array}\right. , & x\in\Omega, t\in(0,T] \\ \frac{\partial v}{\partial \nu} = 0, & x\in\partial\Omega, t\in(0,T] \\ v(x,0) = v^0(x), h(x,0) = h^0(x), & x\in\Omega. \end{array}\right. \tag{0.1}$$

This is a ionic model involving two ionic currents (the second and the third term in the right hand side of the first equation). Here, $\Omega\subset\mathbb{R}^3$ is an open, bounded and Lipschitz domain occupied by the heart, v is the *transmembrane electric potential*, h is the *gating variable* modelling the potassium gate regulating the action potential duration and ν is the outward normal vector to $\partial\Omega$. The parameters τ_{in}, τ_{out}, τ_{open} and τ_{close} are four positive time constants for the inward sodium current, outside potassium current and for the gating variable. The stimulus current J_{stim} is an external current applied in brief pulses by the experimenter. We will also consider the Fenton-Karma model [3] involving three ionic currents corresponding to sodium, calcium and potassium, with two gating variables f and s (f/s standing for *fast/slow*):

$$\left\{\begin{array}{ll} v_t = \triangle v - (J_{fast} + J_{slow} + J_{ung}) + J_{stim}, & x\in\Omega, t\in(0,T] \\ f_t = \left\{\begin{array}{ll} -\frac{f}{\tau_{fclose}}, & v > v_{fgate}, \\ \frac{1-f}{\tau_{fopen}}, & v < v_{fgate} \end{array}\right. , & x\in\overline{\Omega}, t\in(0,T] \\ s_t = \left\{\begin{array}{ll} -\frac{s}{\tau_{sclose}}, & v > v_{sgate}, \\ \frac{1-s}{\tau_{sopen}}, & v < v_{sgate} \end{array}\right. , & x\in\overline{\Omega}, t\in(0,T] \\ \frac{\partial v}{\partial \nu} = 0, & x\in\partial\Omega, t\in(0,T] \\ v(x,0) = v^0(x),\ f(x,0) = f^0(x),\ s(x,0) = s^0(x), & x\in\Omega. \end{array}\right. \tag{0.2}$$

The *fast inward current* J_{fast} takes the form

$$J_{fast} := -\frac{fQ(v)}{\tau_{fast}},\ \text{where } Q(v) := \left\{\begin{array}{ll} (v-v^*)(1-v), & v > v^* \\ 0, & v \le v^* \end{array}\right.$$

and τ_{fast} is the *characteristic time* for this current. It is responsible for the depolarization of the membrane and only depends on the inactivation-reactivation gate f, i.e., it inactivates this current after depolarization an activates it after re-polarization. The *slow inward current* J_{slow} takes the form

$$J_{slow} := -\frac{s\sigma(v)}{\tau_{slow}},\ \text{where } \sigma(v) := \frac{1+\tanh(k(v-v_\sigma))}{2}.$$

The *ungated (slow outward) current* J_{ung} is a piecewise linear function, is responsible for the re-polarization of the membrane and takes the explicit form

$$J_{ung} := P(v),\ \text{where } P(v) := \left\{\begin{array}{ll} \frac{1}{\tau_r}, & v > v^* \\ \frac{v}{\tau_{out}}, & v \le v^*. \end{array}\right.$$

In the first part of the presentation, we will consider the well-posedness problem associated to these two models. For the existence part, we will apply a Faedo-Galerkin technique to construct approximate solutions [5]. The main difficulty with respect to other models appearing in cardiology and involving the same kind of nonlinearity in the equation of v (bidomain or Fitz-Hugh Nagumo models [2]) is the presence of the discontinuous coefficients with respect to v in the gating equations verified by h, f or s leading to ODEs systems with discontinuous nonlinearities for which the appropriate solutions are those of Filippov type [1], [4], so that the gating equations will be interpreted as differential inclusions. The last part of the presentation will be devoted to some control problems associated to these to models.

References

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