Balancing Transformations for Infinite-Dimensional Systems

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We consider model reduction by balanced truncation for infinite-dimensional linear systems. The goal is to apply balancing and truncation to L^2-well-posed linear system with a nuclear Hankel operator in order to obtain a finite dimensional system approximating the input-output behavior in the H_∞ norm.

Our main result is that we explicitly construct pseudo-similarity transformations from factors of the gramians in analogy to the finite dimensional transformations in [3]. These possibly unbounded transformations can be applied to the generators (A, B, C) of the system and yield a new well-posed system on ℓ_2, which is balanced in the sense that its gramians are both equal to a diagonal operator on ℓ_2. This also generalizes results on ℓ_2 balanced realizations from [1] to a larger class of systems. Subsequently, a balanced realization may be truncated, making an error which is bounded by the Hankel singular values as recently proved by [2]. Moreover, we consider reduced transformations which yield the truncated system directly. It should be mentioned that we do not need any controllability or observability assumptions for our approach.

References

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[3] M.S. Tombs and I. Postlethwaite. Truncated balanced realization of a stable non-minimal state-space system. *Int. J. Control*, 46(4):1319–1330, 1987.

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