Poster Proposal: Feedback Stabilization of a Thermal Fluid with Application to Control Design of Energy Efficient Buildings

Weiwei Hu *

The problems of designing and controlling energy efficient building systems are naturally described as distributed parameter optimization and control problems with boundary inputs. Moreover, new low energy concepts such as chilled beams and radiant heating lead to problems with Dirichlet, Neumann and Robin type boundary conditions. Thermal fluid dynamics that describe the physics of displacement ventilation and buoyancy-driven flows are modeled by the Boussinesq approximations. In the design of energy efficient buildings, it is natural to consider control formulations that account for minimizing energy consumption, provide reasonable performance and lead to control laws that are robust with respect to disturbances. These issues lead naturally to Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG) and MinMax control designs since these approaches can be employed to address performance, robustness and energy consumption. In addition, these methods allow one to use the form of the control laws to provide guidance in addressing design issues such as where to "optimally" place sensors and actuators (vents, heating and cooling strips, etc.).

In order to deal with a room level problem, we model the thermal flow by the Boussinesq equations (i.e the coupled Navier-Stokes/heat equation) and investigate basic control theory questions for this system. I first establish that it is possible to locally exponentially stabilize the nonlinear Boussinesq equations around a steady-state thermal-flow by applying finite dimensional Neumann/Robin type boundary control. The proof is valid in two and three spatial dimensions. The feedback controller is obtained by solving a LQR problem for the linearized Bounssinesq equations. It can be shown that when this approach is feasible, the Riccati-based optimal control provides a local stabilizing feedback controller for the full nonlinear Boussinesq equations. Furthermore, representation theory can be used to provide specific integral forms of the feedback control laws which are essential to the development of computational algorithms and helpful in dealing with optimal sensor/actuator placement problems.

After the theoretical foundations are completed, a finite element Galerkin approximation is developed to Boussinesq system (based on Taylor-Hood elements) and it can be shown that this algorithm produces convergent control laws. Finally, this algorithm is used to conduct several numerical experiments on a 2D problem to illustrate the theory and demonstrate the computational algorithm. In order to avoid the so called Compatibility Condition for Dirichlet boundary control in 3D problems, a scaled Robin boundary condition is used to approximate the Dirichlet boundary condition and tested the resulting numerical algorithm on several problems.

*Department of Mathematics, University of Southern California, Los Angeles, CA 90089-2532, USA. E-mail: weiweihu@usc.edu, Telephone: (213)740-3755.