

University of Craiova  
Faculty of Sciences  
Department of Mathematics and Doctoral School of Sciences

WORKSHOP  
APPLIED MATHEMATICS: METHODS  
AND MODELING  
Craiova, May 7-8, 2017

**BOOK OF ABSTRACTS**

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# PLENARY TALK (PT)

# Variational analysis and optimal control of two-dimensional contact problems

Mircea Sofonea

University of Perpignan Via Domitia, France

sofonea@univ-perp.fr

**Abstract:** The interest in contact problems involving thin structures such as beams, bars and plates lies in the fact that their mathematical analysis avoids some of the complications arising in 3D settings and often provides insight to possible types of behaviour of the solution. On the other hand, such structures abound in all branches of engineering and so there is intrinsic interest in these models, too. Finally, these models allow for faster and more comprehensive computer simulations. In this lecture we consider two models of contact with unilateral constraints for a 2D bar. Our main interest lies in the fact that, since the bar is long and thin, the models are found to have two dependent variables: the vertical displacement which describes the motion of the central axis of the bar, and the horizontal displacement which takes tangential shear into account. The first model we consider is static and frictionless. There, the contact is described with both normal compliance and Signorini's contact conditions. We describe the problem and derive its weak formulation, which is in the form of an elliptic variational inequality of the first kind. Next, we establish the existence of a unique weak solution to the model. We also prove the continuous dependence of the weak solution with respect to the applied tractions and constraints. We proceed with the study of an associated control problem for which we prove the existence of an optimal pair. The second model is quasistatic and frictional. There, the contact is with normal compliance and is associated to the classical version of Coulomb's law of dry friction. The variational formulation of the problem is in a form of an evolutionary quasivariational inequality. We prove the weak solvability of the problem, then we provide numerical simulations together with various mechanical interpretations.

APPLIED MATHEMATICS IN INDUSTRY  
(AMI)

**EXPERIMENTAL RESEARCHES FOR AGEING CURVES  
DETERMINATION OF 6061 ALUMINUM ALLOY**

**Marin PETRE, Ion BUGA, Gheorghe POPA**

ALRO, Milcov Street, No. 1, Slatina 230077, Romania

mapetre@alro.ro

**Abstract:** The ternary Al-Mg-Si alloys forms the 6xxx series aluminum alloys which from commercial point of view are the most frequently used due to the ease of processability and for the mechanical properties which they acquire during thermal treatment. The understanding and control of precipitation during artificial aging is critical for achieving optimum mechanical properties in 6xxx alloys. The objectives of this paper are determination and optimization of ageing curves of 6061 alloy. Therefore, an experimental program was conceived to allow investigation of the mechanical properties for various cycles of heat treatment.

SPECIAL SESSION: Mathematical modeling  
in contact mechanics (SS)

## History-dependent contact problems: numerical approximation using penalization and regularization methods

Flavius Pătrulescu

*Tiberiu Popoviciu* Institute of Numerical Analysis, Romanian Academy, Cluj-Napoca

fpatrulescu@ictp.acad.ro

**Abstract:** We consider some recent mathematical models in Contact Mechanics. The process is quasistatic and the contact is modelled with new and nonstandard conditions which involve memory effects. We provide the unique weak solvability using arguments of history-dependent variational inequalities. Moreover, we apply the penalization and regularization methods to approximate the weak solution. Finally, we give some numerical simulations.

## Partial differential inclusions in reflexive Orlicz-Sobolev spaces

Nicușor Costea

Department of Mathematics and Computer Science, POLITEHNICA University of Bucharest,  
313 Splaiul Independenței, 060042 Bucharest, Romania

nicusorcostea@yahoo.com    nicusor.costea@mathem.pub.ro

**Abstract:** We study the weak solvability of PDI's of the type

$$-\operatorname{div}(a(|\nabla u|)\nabla u) \in \partial_C f(x, u(x)), \text{ in } \Omega,$$

subject to Dirichlet boundary condition in a domain  $\Omega \subset \mathbb{R}^N$  with Lipschitz boundary  $\partial\Omega$ . Here,  $a : (0, \infty) \rightarrow (0, \infty)$  is such that

$$\Phi(t) = \int_0^t a(s)s \, ds,$$

defines an  $N$ -function and the corresponding Orlicz-Sobolev space  $W_0^1 L^\Phi(\Omega)$  is reflexive. The function  $f : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$  is locally Lipschitz w.r.t. the second variable and  $\partial_C$  denotes the Clarke subdifferential of  $t \mapsto f(x, t)$ .

Using a minimization technique and the Zero Altitude Mountain Pass Theorem for locally Lipschitz functionals the existence of at least one weak solution is established. A multiplicity

alternative is also proved via nonsmooth Schechter theory. More precisely, we show that either the problem possesses at least two nontrivial weak solutions or a rich family of negative eigenvalues.

This is a joint work with Csaba Varga (Babes-Bolyai University, Cluj-Napoca) and Gheorghe Moroşanu (Central European University, Budapest).

## **On the weak solvability of an antiplane contact problem for nonlinearly elastic materials**

**Mariana Florentina Chivu**

Doctoral School of Sciences, University of Craiova, A.I. Cuza 13, 200585, Craiova, Romania

chivumarianaflorentina@yahoo.com

**Abstract:** We consider an antiplane frictional contact problem for nonlinearly elastic materials. We introduce the notion of weak solution as a solution of a coupled variational system consisting of a variational equation and a variational inequality. Then, we are interested to prove the existence and the uniqueness of the weak solution. Also, a Hölder continuous dependence on the data is discussed. (based on a joint work with A. Matei)

## **On the boundary optimal control of a nonlinear problem arising from contact mechanics**

**Constantin Niţă**

Doctoral School of Sciences, University of Craiova

nita\_gyk@yahoo.com

**Abstract:** A boundary value problem which describes a frictional antiplane contact model is considered. Firstly we discuss its well-posedness. Then, we study the boundary optimal control: we prove the existence of at least one optimal pair and we deliver an optimality condition (based on a joint work with N. Cîndea, A. Matei and S. Micu).

SESSION  
YOUNG RESEARCHERS TO  
EXPERIENCED RESEARCHERS.

Let's meet!

(YRER)

(abstracts in the authors' alphabetical order )

## No-flux boundary problems with variable exponents

Maria-Magdalena Boureanu

Department of Applied Mathematics, University of Craiova, Romania

mmboureanu@yahoo.com

**Abstract:** The variable exponent partial differential equations represent the topic of many studies lately, and the interest manifested towards them comes also from the large range of possible applications, such those concerning electrorheological fluids, thermorheological fluids, elastic materials, image restoration, mathematical biology etc. At the same time, the so-called "no-flux" problems make the subject of various recent studies because the no-flux boundaries represent surfaces that are impermeable to some contaminants and such surfaces are quite common in nature. Our interest incorporates the two directions of research, and we base our work on the critical point theory and on variational arguments.

## Fractional models for heat propagation. Applications in fusion plasma physics

Dana Constantinescu

Department of Applied Mathematics, University of Craiova, Romania

constantinescu.dana@ucv.ro

**Abstract:** After a short review of basic information on fractional calculus, the fractional diffusion and heat equations are derived. It is shown that these fractional models involve memory effects (through the fractional time derivative) and non-local spatial effects (through fractional spatial derivatives). The practical importance of the theoretical enlargement is also discussed.

In order to show the usefulness of fractional models we focus on the description of heat propagation during thermo-controlled nuclear reactions. These reactions occur in experiments made in tokamaks (toroidal devices) in order to get energy through nuclear fusion. In this particular case the heat propagation is correlated with the existence of a 3D magnetic field which is strongly anisotropic. Some analytical considerations are made for models involving simple magnetic fields and specific numerical methods are used for solving complex situations. The obtained results point out the dependence of heat propagation on the ideal (unperturbed magnetic field) and on its perturbations.

# Multistability, self-excited and hidden attractors in some 3D dynamical systems

Raluca Efrem

Department of Mathematics, Faculty of Sciences, University of Craiova, Romania

ra.efrem@yahoo.com

**Abstract:** The coexistence of many attractors (multistability) and the identification of hidden attractors is analyzed in some 3D dynamical systems of interest in physics. The deep analysis of multistable systems is very important in applications, because it is interesting to precise the conditions for a system to remain on the most desirable attractor and to reduce the risk of a sudden jump to undesired behavior.

We focus on Lorenz-like systems (with applications in the study of convective fluid motion), Chua system (with application in the study of electrical circuits) and a low dimensional system describing the variation of plasma parameters in fusion plasma physics. For these systems self-excited attractors (attractors whose basin of attraction intersects any neighborhood of an unstable equilibrium) are localized numerically by standard computational procedures and hidden attractors (whose basin of attraction is located far away from equilibrium points) are pointed out from numerical simulations. In each case some specific analytical-numerical procedures are developed in order to localize them and to describe their basin of attraction. The obtained results are interpreted in terms of physical phenomena described by the dynamical systems, which gives a better understand of our mathematical considerations.

## Perturbed fractional eigenvalue problems

Maria Fărcășeanu

Department of Mathematics, University of Craiova, 200585 Craiova, Romania

farcaseanu.maria@yahoo.com

**Abstract:** Let  $\Omega \subset \mathbb{R}^N$  ( $N \geq 2$ ) be a bounded domain with Lipschitz boundary. For each  $p \in (1, \infty)$  and  $s \in (0, 1)$ , we denote by  $(-\Delta_p)^s$  the fractional  $(s, p)$ -Laplacian operator. We study the existence of nontrivial solutions for a perturbation of the eigenvalue problem  $(-\Delta_p)^s u = \lambda |u|^{p-2} u$ , in  $\Omega$ ,  $u = 0$ , in  $\mathbb{R}^N \setminus \Omega$ , with a fractional  $(t, q)$ -Laplacian operator in the left-hand side of the equation, when  $t \in (0, 1)$  and  $q \in (1, \infty)$  are such that  $s - N/p = t - N/q$ .

We show that nontrivial solutions for the perturbed eigenvalue problem exists if and only if parameter  $\lambda$  is strictly larger than the first eigenvalue of the  $(s, p)$ -Laplacian. This talk is based on some recent results obtained in collaboration with Mihai Mihăilescu and Denisa Stancu-Dumitru.

## **Influence of parameters in the feedback linearization for a mixing flow model**

**Adela Ionescu**  
University of Craiova  
adela0404@yahoo.com

**Abstract.** This paper exhibits some recent achievements in the qualitative analysis of the mixing flow model. For 2d case, the parameter influence in the feedback linearization method is taken into account, for a logistic-type perturbation of the model. The results will be used for further comparative analysis of the mixing models.

## **About symmetries, pseudosymmetries and conservation laws in Lagrangian and Hamiltonian $k$ -symplectic formalisms**

**Florian Munteanu**

Department of Applied Mathematics, University of Craiova  
Al.I. Cuza 13, Craiova 200585, Romania

munteanufm@central.ucv.ro

**Abstract:** The  $k$ -symplectic formalism is a generalization to field theories of the standard symplectic formalism in autonomous Mechanics, which is the geometric framework for describing autonomous dynamical systems. This formalism is based on the polysymplectic formalism developed by Günther in 1987. The  $k$ -Symplectic Geometry provides the simplest geometric framework for describing certain class of first-order classical field theories. The purpose of this talk is to present Lagrangian and Hamiltonian  $k$ -symplectic formalisms. Further, we will recall the notions of symmetry, conservation law and relationship between this in the framework of  $k$ -symplectic geometry and we will introduce the notion of pseudosymmetry as a natural extension of symmetry. Without the help of a Noether type theorem, using only symmetries and pseudosymmetries, we will obtain new kinds of conservation laws for  $k$ -symplectic Hamiltonian systems and  $k$ -symplectic Lagrangian systems.

## On nonholonomic higher order dynamics

Marcela Popescu and Paul Popescu

University of Craiova, Department of Applied Mathematics

marcelacpopescu@yahoo.com and paul\_p\_popescu@yahoo.com

**Abstract:** Dynamics associated with higher order lagrangians and hamiltonians are studied, in the presence of linear, affine and non-linear constraints of an arbitrary order that are adapted to a foliation  $\mathcal{F}$  on a manifold  $M$ . According to the degree of the Euler-Lagrange equation of the higher order lagrangian, we study several cases, when some regularity conditions can be considered. The regularity conditions considered here are related to some semispray like conditions, when certain vector fields are obtained having as integral curves solutions of the motion. Continuous and discrete cases are both considered.

## Two sequences of solutions for indefinite superlinear-sublinear elliptic equation involving nonlinear Neumann boundary condition

Ionela-Loredana Stăncuț

University of Craiova

stancutloredana@yahoo.com

**Abstract:** We study the existence of infinitely many solutions of a nonlinear Neumann problem of the following type:

$$-\operatorname{div}(|\nabla u|^{m(x)-2}\nabla u) + |u|^{m(x)-2}u = f(x, u) \text{ in } \Omega, \quad |\nabla u|^{m(x)-2} \frac{\partial u}{\partial \nu} = g(x, u) \text{ on } \partial\Omega,$$

where  $\Omega$  is a bounded domain in  $\mathbb{R}^N$  with smooth boundary  $\partial\Omega$ , the functions  $f(x, u)$  and  $g(x, u)$  are continuous on  $\bar{\Omega} \times \mathbb{R}$  and on  $\partial\Omega \times \mathbb{R}$ , respectively, and odd with respect to  $u$ , while  $m$  is a Lipschitz continuous function. More specifically, we study the existence of a sequence of solutions diverging to infinity provided that the nonlinear term is locally superlinear and the existence of a sequence of solutions converging to zero provided that the nonlinear term is locally sublinear.