Workshop APPLIED MATHEMATICS: METHODS AND MODELING
Craiova, May 7 - 8, 2017

Variational analysis and optimal control of two-dimensional contact problems

Mircea Sofonea

University of Perpignan Via Domitia, France

sofonea@univ-perp.fr

Abstract: The interest in contact problems involving thin structures such as beams, bars and plates lies in the fact that their mathematical analysis avoids some of the complications arising in 3D settings and often provides insight to possible types of behavior of the solution. On the other hand, such structures abound in all branches of engineering and so there is intrinsic interest in these models, too. Finally, these models allow for faster and more comprehensive computer simulations. In this lecture we consider two models of contact with unilateral constraints for a 2D bar. Our main interest lies in the fact that, since the bar is long and thin, the models are found to have two dependent variables: the vertical displacement which describes the motion of the central axis of the bar, and the horizontal displacement which takes tangential shear into account. The first model we consider is static and frictionless. There, the contact is described with both normal compliance and Signorini's contact conditions. We describe the problem and derive its weak formulation, which is in the form of an elliptic variational inequality of the first kind. Next, we establish the existence of a unique weak solution to the model. We also prove the continuous dependence of the weak solution with respect to the applied tractions and constraints. We proceed with the study of an associated control problem for which we prove the existence of an optimal pair. The second model is quasistatic and frictional. There, the contact is with normal compliance and is associated to the classical version of Coulomb's law of dry friction. The variational formulation of the problem is in a form of an evolutionary quasivariational inequality. We prove the weak solvability of the problem, then we provide numerical simulations together with various mechanical interpretations.