FINAL SCIENTIFIC REPORT
(05.10.2011-04.10.2014)

- Short description of the research project (general data)

  - **The title: Strongly Nonlinear Problems in Contact Mechanics**
  - **GRANT of the Romanian National Autority for Scientific Research, CNCS-UEFISCDI**
  - **Cod: PN-II-RU-TE-2011-3-0223**
  - **05.10.2011-04.10.2014**
  - The overall goal of the project is to improve the understanding of real-world problems governed by Partial Differential Equations in Contact Mechanics. We focus on the existence, uniqueness and stability of the weak solutions; also, the efficient approximation of the weak solutions and optimal control on the boundary for some contact models, are of interest for us. To achieve these targets, new trends in Advanced Applied Mathematics are required, combining Mechanics of Continua, Contact Mechanics and Mechanics of Materials with mathematical areas as PDEs, Nonlinear Analysis, Convex Analysis and Numerical Analysis.
  - **Team**

    Andaluzia-Cristina Matei (**Director**)

    Maria Magdalena Boureanu

    Ionel Roventa

  - **Main objectives**

(O1) The first objective concerns alternative variational approaches in the mathematical treatment of contact problems, including the approach with dual Lagrange multipliers and the approach via bipotentials.

(O2) The second objective focuses on qualitative properties in the variational study of new contact models for non-standard materials such as materials whose constitutive laws involve terms with variable exponent, materials with coupled properties, viscoelastic materials.

- The **main results** were included in 20 works as follows.

Published articles

1. M. Barboteu**, A. Matei** and M. Sofonea, On the behavior of the solution of a viscoplastic contact problem, Quarterly of Applied Mathematics (ISI), DOI: http://dx.doi.org/10.1090/S0033-569X-2014-01345-4, published online September 25, 2014.

2. A. Matei, Weak Solutions via Lagrange Multipliers for a Slip-dependent Frictional Contact Model, IAENG International Journal of Applied Mathematics, 44 (3), 2014, 151-156 (special issue WCE 2014-ICAEM'14);
http://www.iaeng.org/IJAM/issues_v44/issue_3/index.html)

3. A. Matei, Weak solvability via Lagrange multipliers for contact problems involving multi-contact zones, Mathematics and Mechanics of Solids (ISI);
DOI: 10.1177/1081286514541577, published online July 7, 2014.

4. A. Matei, An existence result for a mixed variational problem arising from Contact Mechanics, Nonlinear Analysis Series B: Real World Application (ISI), 20 (2014), 74-81.
DOI: 10.1016/j.nonrwa.2014.01.010.

5. A. Matei, A variational approach via bipotentials for a class of frictional contact problems, Acta Applicandae Mathematicae (ISI), DOI: 10.1007/s10440-014-9868-1, published online February 7, 2014.

6. I. Roventa, Generalized equilibrium problems related to Ky Fan inequalities, Abstract and Applied Analysis, (ISI), Volume 2014, Article ID 301901, 6 pages; http://dx.doi.org/10.1155/2014/301901.

7. M. Boureanu, A. Matei and M. Sofonea, Nonlinear problems with p(.)-growth conditions and applications to antiplane contact models, Advanced Nonlinear Studies (ISI), 14 (2014), 295-313.

8. A. Matei, On the solvability of mixed variational problems with solution-dependent sets of Lagrange multipliers, Proceedings of The Royal Society of Edinburgh, Section: A Mathematics (ISI); 143(05), October 2013, 1047-1059;
http://dx.doi.org/10.1017/S0308210512000637; ISSN: 0308-2105.

9. S. Hüeber, **A. Matei**, B. Wohlmuth, A contact problem for electro-elastic materials, Journal of Applied Mathematics and Mechanics (ZAMM) (ISI), DOI: 10.1002/zamm.201200235, 93 (10-11), 789-800, October 2013. Special Issue: Mathematical Modeling: Contact Mechanics, Phase Transitions, Multiscale Problems.

10. A. Matei, Weak solvability via Lagrange multipliers for two frictional contact models, Proceedings of 11-th French-Romanian Conference on Applied Mathematics (Colloque Franco-Roumain), 2012, Bucharest, Annals of the University of Bucharest (mathematical series), 4(LXII), 179-191, 2013.

11. A. Matei, A variational approach via bipotentials for unilateral contact problems, Journal of Mathematical Analysis and Applications (ISI) , ISSN 0022-247X, Volume 397, Issue 1, 1 January 2013, Pages 371-380. http://dx.doi.org/10.1016/j.jmaa.2012.07.065.

12. I. Andrei, N. Costea and **A. Matei**, Antiplane shear deformation of piezoelectric bodies in contact with a conductive support, Journal of Global Optimization (ISI); ISSN: 0925-5001 DOI: 10.1007/s10898-011-9815-x; Volume 56, Issue 1, pp 103-119, May 2013.

13. M. Barboteu, **A. Matei** and M. Sofonea, Analysis of Quasistatic Viscoplastic Contact Problems with Normal Compliance, The Quarterly Journal of Mechanics and Applied Mathematics (ISI) , DOI: 10.1093/qjmam/hbs016, 65(4), 555-579, 2012, ISSN 0033-5614.

14. I. Roventa, A note on Schur-concave functions, Journal of Inequalities and Applications (ISI) , DOI: 10.1186/1029-242X-2012-159, 2012:159, 9 pages.

15. M.M. Boureanu, Remarks on Neumann boundary value problems with variable exponents, Bulletin of the Transilvania University of Brasov, Series III: Mathematics, Informatics, Physics, 5(54), 55-66, 2012.

Published research monograph

16. M. Sofonea and **A. Matei**, Mathematical Models in Contact Mechanics, London Mathematical Society, Lecture Note Series 398, Cambridge University Press, 2012 (research monograph).

Accepted articles

17. A. Matei, Two abstract mixed variational problems and applications in Contact Mechanics, Nonlinear Analysis*:* Real World Applications, accepted in September 2014.

Published conference paper

18. A. Matei, A mixed variational formulation for a slip-dependent frictional contact model, Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2014, 2-4 July, 2014, London, U.K., pp 750-754 (ISBN: 978-988-19253-5-0, ISSN: 2078-0958).

Submitted articles

19. A. Matei, Weak solutions via Lagrange multipliers for contact models with normal compliance, special issue IECMSA 2014, Konuralp Journal of Mathematics submitted.

20. I. Roventa, Strongly majorization properties and applications related to Schur-convexity, submitted.

- An important component of the documentation-research activity was realized during the following **research visits**:

    - Technische Universitat Munchen (TUM), Mathematik und Informatik Zentrum: August 10- August 18, 2014 (**A. Matei**)
      We focused on a class of evolutionary problems in Contact Mechanics via techniques with Lagrange multipliers.
    - Technische Universitat Munchen (TUM), Mathematik und Informatik Zentrum: August 26- September 12, 2013 (**A. Matei**)
      We focused on a class of contact problems with normal compliance.
    - Technische Universitat Munchen (TUM), Mathematik und Informatik Zentrum: July 22- August 02, 2013 (**A. Matei**)
      We focused on a class of viscoelastic contact problems.
    - University of Perpignan (LAMPS): June 10-23, 2012 (**A. Matei**)
      We focused a class of viscoplastic mathematical models in contact mechanics.
    - Technische Universitat Munchen (TUM), Mathematik und Informatik Zentrum: April 19-30, 2012 (**A. Matei**)
      We focused on a class of electro-elastic problems.
    - Milano Bicocca University: May 2-11, 2012 (**M.M. Boureanu)**
      We focused on a class of nonlinear PDEs.

- A part of the results were disseminated at the following events:

    - The 2014 International Conference of Applied and Engineering Mathematics London, U.K., 2-4 July (ICAEM'14), into the frame of The World Congress on Engineering 2014 (WCE 2014) Imperial College London, U.K. **(A. Matei)**
    - The 10-th AIMS Conference on Dynamical Systems, Differential Equations and Applications, July 07- July 11, 2014, Madrid, Spain **(M. Boureanu)**
    - 3-rd International Eurasian Conference on Mathematical Sciences and Applications, Technische Universitat Wien (TU Vienna), Austria, 25-28 August, 2014 **(A. Matei)**
    - The 21-st Conference of Applied and Industrial Mathematics-CAIM 2013, 19-22 September, Bucharest, Romania (**A. Matei**)
    - Workshop for Young Researchers in Mathematics, May 09-10, 2013 Ovidius University, Constanta, Romania (**A. Matei**).
    - XI-eme Colloque Franco-Roumain de Mathematiques Appliquees, Universite de Bucarest, 24-30 Aout 2012, Roumanie (**A. Matei**: joint work with Mircea Sofonea)
    - 41-eme Congres National d'Analyse Numerique, SuperBesse- Puy-de-Dome, 21-25 mai 2012, Universite Blaise Pascal, Clermont-Ferrand, France (**joint work A. Matei and I. Roventa**).
    - Workshop for Young Researchers in Mathematics, May 10-11, 2012, Ovidius University, Constanta, Romania (**A. Matei**).

- Our scientific seminar hosted the following **invited talk**:

History-dependent operators in Contact Mechanics
Professor Mircea Sofonea, University of Perpignan, France; October 12, 2012.

- Short description of the main results

  - The results corresponding to the 1-th objective

    - **A. Matei**, Two abstract mixed variational problems and applications in Contact Mechanics, Nonlinear Analysis: Real World Applications, **ISI,** accepted September 2014.

Abstract - We consider two abstract mixed variational problems. Each of them consists of a system of two variational inequalities. The first problem involves two convex functionals while the second one involves a convex functional and a bifunctional which depends on a Lagrange multiplier in the first argument and is convex in the second argument. We obtain existence and uniqueness results for the first problem. Then, we combine the results we get with a fixed point technique in order to investigate the existence and the uniqueness of the solution of the second problem. The abstract results we obtain can be applied to the weak solvability of frictional contact models for nonlinearly elastic materials. To illustrate the applicability, three examples of frictional contact models are discussed.

    - **A. Matei**, Weak Solutions via Lagrange Multipliers for a Slip-dependent Frictional Contact Model**,** IAENG International Journal of Applied Mathematics, 44 (3), 2014, 151-156 (special issue, http://www.iaeng.org/IJAM/issues_v44/issue_3/index.html).

We consider a 3D elastostatic slip-dependent frictional contact problem which consists of a system of partial differential equations associated with a homogeneous displacement boundary condition, a traction boundary condition and a frictional contact boundary condition involving a slip-dependent friction bound. After describing the mechanical model, we deliver a variational formulation as a mixed variational problem whose Lagrange multipliers set is solution-dependent. Then, the existence and the boundedness of the solutions are investigated. The proof is based on a recent result for an abstract mixed variational problem with solution-dependent set of Lagrange multipliers.This work is the extended and the improved version of the conference paper ICAEM'14 (WCE 2014):

[A. Matei, A mixed variational formulation for a slip-dependent frictional contact model, Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2014, 2-4 July, 2014, London, U.K., pp 750-754 (ISBN: 978-988-19253-5-0, ISSN: 2078-0958).]

    - **A. Matei,** Weak solutions via Lagrange multipliers for frictional contact models with normal compliance, special issue IECMSA 2014, Konuralp Journal of Mathematics**,** Proceedings of 3rd International Eurasian Conference on Mathematical Sciences and Applications**,** submitted.

Abstract - We consider a 3D elastostatic frictional contact problem with normal compliance, which consists of a systems of partial differential equations associated with a displacement boundary condition, a traction boundary condition and a frictional contact boundary condition. The frictional contact is modeled by means of a normal compliance condition and a version of Coulomb's law of dry friction. After we state the problem and the hypotheses, we deliver a variational formulation as a mixed variational problem with solution-dependent Lagrange multipliers set. Next, we prove the existence and the boundedness of the weak solutions based on recents abstract results we have got in the paper[A. Matei, On the solvability of mixed variational problems with solution-dependent sets of Lagrange multipliers, Proceedings of The Royal Society of Edinburgh, Section: A Mathematics, 143(05), 2013, 1047-1059
]

- **A. Matei,** Weak solvability via Lagrange multipliers for contact problems involving multi-contact zones, Mathematics and Mechanics of Solids, ISI , DOI: 10.1177/1081286514541577.

Abstract - We investigate the behavior of an elastic body which is in frictional contact with a foundation on a part of the boundary and, on another part of the boundary it can comes in contact with a rigid obstacle. We associate this physical setting with two mechanical models. Every model is mathematically described by a boundary value problem which consists of a systems of partial differential equations associated with a displacement condition, a traction condition, a frictional contact condition and a frictionless unilateral contact condition. In both models the unilateral contact is described by Signorini's condition with non zero gap. The difference between the models is given by the friction condition we use. In the first model we use a condition with prescribed normal stress. In the second one, we use a frictional bilateral contact condition. For every model, we deliver a variational formulation which is a generalized saddle point problem. Then, we investigate the existence, the uniqueness and the boundedness of the weak solutions. The approximation of the weak solutions is also discussed. (initial title " A variational technique for solving a class of multi-contact problems *").*

- **A. Matei**, An existence result for a mixed variational problem arising from Contact Mechanics, Nonlinear Analysis: Real World Applications (ISI), vol. 20, December 2014, 74-81, DOI:10.1016/j.nonrwa.2014.01.010.

Abstract - We consider a mixed variational problem involving a nonlinear, hemicontinuous, generalized monotone operator. The proposed problem consists of a variational equation in a real reflexive Banach space and a variational inequality in a subset of a second real reflexive Banach space. We investigate the existence of the solution using a fixed point theorem for set valued mapping. An example arising from Contact Mechanics illustrates the theory.

- **I. Roventa**, Generalized equilibrium problems related to Ky Fan inequalities, Abstract and Applied Analysis Volume 2014 (2014**),** ISI, Article ID 301901, 6 pages http://dx.doi.org/10.1155/2014/301901.

Abstract - We study a generalized equilibrium problem by using a non symmetric extension of Ky Fan inequality. As an application, we present a fixed point type algorithm inspired by a model from A. Tada and W. Takahashi, Weak and strong convergence theorems for a nonexpansive mapping and an equilibrium problem, J. Optim. Theory Appl. 133 (2007), 359-370.

- **A. Matei**, A variational approach via bipotentials for a class of frictional contact problems, Acta Applicandae Mathematicae (ISI), DOI: 10.1007/s10440-014-9868-1.

Abstract - We study a class of frictional contact problems with prescribed normal stress, for non-linearly elastic materials. Using a bipotential which depends on the constitutive map and its Fenchel conjugate, and a potential which depends on the prescribed normal stress and the coefficient of friction, we deliver a weak formulation which consists of a system of two variational inequalities. The unknown is the pair of displacement vector and Cauchy stress tensor. We prove the existence and the uniqueness of the weak solution by using minimization arguments. We also discuss some connections of the new variational approach to previous variational approaches.

- **A. Matei**, On the solvability of mixed variational problems with solution-dependent sets of Lagrange multipliers, Proceedings of The Royal Society of Edinburgh, Section: A Mathematics (ISI); 143(05), October 2013, 1047-1059;
  http://dx.doi.org/10.1017/S0308210512000637; ISSN: 0308-2105.

Abstract - We study an abstract mixed variational problem, the set of the Lagrange multipliers being dependent on the solution. The problem consists of a system of a variational equation and a variational inequality. We prove the existence of the solution based on a fixed point technique for weakly sequentially continuous maps. Next, we apply the abstract result to the weak solvability of a boundary value problem which models the frictional contact between a cylindrical deformable body and a rigid foundation.

- **A. Matei**, Weak solvability via Lagrange multipliers for two frictional contact models, Proceedings of 11-th French-Romanian Conference on Applied Mathematics (Colloque Franco-Roumain), 2012, Bucharest, Annals of the University of Bucharest (mathematical series), 4(LXII), 179-191, 2013.

Abstract - We consider two frictional contact models, for nonlinearly elastic materials. For every model, we deliver a weak formulation as a generalized saddle point problem, and then we prove the existence, uniqueness and stability of weak solution. The proofs rely on abstract results in the study of a class of abstract generalized saddle point problems.

- **A. Matei**, A variational approach via bipotentials for unilateral contact problems, Journal of Mathematical Analysis and Applications (ISI), ISSN 0022-247X, Volume 397, Issue 1, 1 January 2013, Pages 371-380. http://dx.doi.org/10.1016/j.jmaa.2012.07.065.

Abstract - We consider a unilateral contact model for nonlinearly elastic materials, under the small deformation hypothesis, for static processes. The contact is modeled with Signorini's condition with zero gap and the friction is neglected on the potential contact zone. The behavior of the material is modeled by a subdifferential inclusion, the constitutive map being proper, convex, and lower semicontinuous. After describing the model, we give a weak formulation using a bipotential which depends on the constitutive map and its Fenchel conjugate. We arrive to a system of two variational inequalities whose unknown is the pair

consisting of the displacement field and the Cauchy stress field. We look for the unknown into a Cartesian product of two nonempty, convex, closed, unbounded subsets of two Hilbert spaces. We prove the existence and the uniqueness of the weak solution based on minimization arguments for appropriate functionals associated with the variational system. How the proposed variational approach is related to previous variational approaches, is discussed too.

- **I. Roventa**, A note on Schur-concave functions, Journal of Inequalities and Applications (ISI), DOI: 10.1186/1029-242X-2012-159, 2012:159, 9 pages.

Abstract - In this paper we consider a class of Schur-concave functions with some measure properties. The isoperimetric inequality and Brunn-Minkowsky's inequality for such kind of functions are presented. Applications in geometric programming and optimization theory are also derived.

- **I. Roventa**, Strongly majorization properties and applications related to Schur-convexity, submitted.

Abstract - In this paper we study some majorization properties with interesting applications in graph theory, optimization theory and geometric inequalities. A strongly notion of majorization is introduced and Hardy-Littlewood-Polya's inequality is generalized. Other results concerning Schur-convex functions are presented.

❖ **The results corresponding to the 2-nd objective**

- M. Barboteu**, A. Matei** and M. Sofonea, On the behavior of the solution of a viscoplastic contact problem, Quarterly of Applied Mathematics (ISI), DOI: http://dx.doi.org/10.1090/S0033-569X-2014-01345-4.

Abstract - We consider a mathematical model which describes the frictionless contact between a viscoplastic body and an obstacle, the so-called foundation. The process is quasistatic and the contact is modeled with normal compliance and unilateral constraint. We provide a mixed variational formulation of the model which involves a dual Lagrange multiplier, then we prove its unique weak solvability. We also prove an estimate which allows us to deduce the continuous dependence of the weak solution with respect to both the normal compliance function and the penetration bound. Finally, we provide a numerical validation of this convergence result.

- **M.M. Boureanu, A. Matei** and M. Sofonea, Nonlinear problems with p(.)-growth conditions and applications to antiplane contact models, Advanced Nonlinear Studies, ISI , ISSN 1536-1365, 14 (2014), 295-313.

Abstract - We consider a general boundary value problem involving operators of the form div(a(·, ∇u(·)) in which a is a Caratheodory function satisfying a p(·)-growth condition. We are interested on the weak solvability of the problem and, to this end, we start by introducing the Lebesgue and Sobolev spaces with variable exponent, together with their main properties. Then, we state and prove our main existence and uniqueness result, Theorem 3.1. The proof is based on a Weierstrass-type argument. We also consider two antiplane contact problems for nonhomogenous elastic materials of Hencky-type. The contact is frictional and it is modelled with a regularized version of Tresca's friction law and a power-law friction, respectively. We prove that the problems cast in the abstract setting, then we use Theorem 3.1 to deduce their unique weak solvability.

- S. Hüeber, **A. Matei**, B. Wohlmuth, A contact problem for electro-elastic materials, Journal of Applied Mathematics and Mechanics (ZAMM) (ISI), DOI: 10.1002/zamm.201200235, 93 (10-11), 789-800, October 2013. Special Issue: Mathematical Modeling: Contact Mechanics, Phase Transitions, Multiscale Problems.

Abstract - We analyze the frictionless unilateral contact between an electro-elastic body and a rigid electrically conductive foundation. On the potential contact zone, we use the Signorini condition with non-zero gap and an electric contact condition with a conductivity depending on the Cauchy vector. We provide a weak variationally consistent formulation and show existence, uniqueness and stability of the solution. Our analysis is based on fixed point techniques for weakly sequentially continuous maps. We conclude by a numerical example that illustrates the applicability of the model.

- I. Andrei, N. Costea and **A. Matei**, Antiplane shear deformation of piezoelectric bodies in contact with a conductive support, Journal of Global Optimization (ISI); ISSN: 0925-5001 DOI: 10.1007/s10898-011-9815-x; Volume 56, Issue 1, pp 103-119, May 2013.

Abstract - We consider a mathematical model which describes the frictional contact between a piezoelectric body and an electrically conductive support.We model the material's behavior with an electro-elastic constitutive law; the frictional contact is described with a boundary condition involving Clarke's generalized gradient and the electrical condition on the contact surface is modelled using the subdifferential of a proper, convex and lower semicontinuous function.We derive a variational formulation of the model and then, using a fixed point theorem for set valued mappings, we prove the existence of at least one weak solution. Finally, the uniqueness of the solution is discussed; the investigation is based on arguments in the theory of variational-hemivariational inequalities.

- M. Barboteu, **A. Matei** and M. Sofonea, Analysis of Quasistatic Viscoplastic Contact Problems with Normal Compliance, The Quarterly Journal of Mechanics and Applied Mathematics (ISI), DOI: 10.1093/qjmam/hbs016, 65(4), 555-579, 2012, ISSN 0033-5614.

Abstract - We consider two quasistatic problems that describe the contact between a viscoplastic body and an obstacle, the so-called foundation. The contact is frictionless and is modelled with normal compliance of such a type that the penetration is not restricted in the first problem, but is restricted with unilateral constraint, in the second one. For each problem we derive a variational formulation, then we prove its unique solvability. The proofs are

based on a recent result on history-dependent quasivariational inequalities obtained in (Sofonea and Matei, Eur. J. Appl. Math. 22 (2011)) . Next, we prove the convergence of the weak solution of the first problem to the weak solution of the second problem, as the stiffness coefficient of the foundation converges to infinity. Finally, we provide a numerical validation of this convergence result. To this end we introduce fully discrete schemes for the numerical approximation of the contact problems, implement them on a computer code and present numerical simulation results in the study of a two-dimensional example.

- **M.M. Boureanu**, Remarks on Neumann boundary value problems with variable exponents, Bulletin of the Transilvania University of Brasov, Series III: Mathematics, Informatics, Physics, 5(54), 55-66, 2012.

Abstract - We are interested in elliptic problems with Neumann boundary conditions that are studied in the framework of isotropic and anisotropic spaces with variable exponents. We establish an existence and a uniqueness result concerning a problem with a general p(.) -Laplace type operator. In addition, we present connections to other results, some of them involving the same operator, some of them involving a general $p^{\rightarrow}(.)$ -Laplace type operator.

- M. Sofonea and **A. Matei**, Mathematical Models in Contact Mechanics, London Mathematical Society, Lecture Note Series 398, Cambridge University Press, 2012 (research monograph).

This text provides a complete introduction to the theory of variational inequalities with emphasis on contact mechanics. It covers existence, uniqueness, and convergence results for variational inequalities, including the modeling and variational analysis of specific frictional contact problems with elastic, viscoelastic, and viscoplastic materials. New models of contact are presented, including contact of piezoelectric materials. Particular attention is paid to the study of history-dependent quasivariational inequalities and to their applications in the study of contact problems with unilateral constraints. The book fully illustrates the cross-fertilization between modeling and applications on the one hand, and nonlinear mathematical analysis on the other. Indeed, the reader will gain an understanding of how new and nonstandard models in contact mechanics lead to new types of variational inequalities and, conversely, how abstract results concerning variational inequalities can be applied to prove the unique solvability of the corresponding contact problems.

Conferences (Abstracts)

- *The International Conference of Applied and Engineering Mathematics London, U.K., 2-4 July 2014 (ICAEM'14), into the frame of The World Congress on Engineering 2014 (WCE 2014) London.*
  - ***A. Matei**, A mixed variational formulation for a slip-dependent frictional contact model, Lecture Notes in Engineering and Computer Science: **Proceedings of The World Congress on Engineering** 2014, 2-4 July, 2014, London, U.K., pp 750-754 (ISBN: 978-988-19253-5-0, ISSN: 2078-0958).*

A 3D slip-dependent frictional contact problem in elastostatics is discussed. We deliver a variational formulation as a mixed variational problem whose Lagrange multipliers set is solution-dependent. Then, the existence and the boundedness of the solutions is investigated. The proof is based on a recent result for an abstract mixed variational problem with

solution-dependent set of Lagrange multipliers obtained in [A. Matei, On the solvability of mixed variational problems with solution-dependent sets of Lagrange multipliers, Proceedings of The Royal Society of Edinburgh, Section: A Mathematics, 143(05), 2013, 1047-1059]. (http://www.iaeng.org/WCE2014/schedule/index.html).

- *The 10th AIMS Conference on Dynamical Systems, Differential Equations and Applications, July 07- July 11, 2014, Madrid, Spain*

  - ***M.M. Boureanu,*** *Variable exponent problems involving generalized operators*

The study of problems with variable exponents is becoming more and more popular due to a wide range of applications to various domains. In this context, we are concerned with elliptic problems involving generalized operators that are related to two well known classes of operators, that is, the Laplace-type operators and the mean curvature-type operators. Our discussion is conducted in the framework of the spaces with variable exponents and the main argumentation is made by means of the critical point theory. (http://www.aimsciences.org/AIMS-Conference/conf-reg2014/abstracts/ss34_Abstract.pdf)

- *3rd International Eurasian Conference on Mathematical Sciences and Applications, Viena, 25-28 August, 2014*

  - ***A. Matei****, Weak solutions via Lagrange multipliers for contact models with normal compliance.*

A 3D elastostatic frictional contact problem with normal compliance was discussed. After stating the problem and the hypotheses, a variational formulation as a mixed variational problem with solution-dependent Lagrange multipliers set was presented. Next, the existence and the boundedness of the weak solutions was justified. (http://www.iecmsa.org/kitapciklar/abstractbook.pdf).

- The 21-st Conference of Applied and Industrial Mathematics-CAIM 2013, 19-22 September, Bucharest, Romania

  - ***A. Matei****, A variational method for solving a class of boundary value problems arising from Contact Mechanics.*

The present work is based on the recent article [A. Matei, On the solvability of mixed variational problems with solution-dependent sets of Lagrange multipliers, Proceedings of The Royal Society of Edinburgh, Section: A Mathematics]. *We focus on a variational method for solving a class of boundary value problems arising from Contact Mechanics. The variational support is an abstract mixed variational problem, the set of the Lagrange multipliers being dependent on the solution. Firstly, we discuss the existence of the solution of the abstract problem. The discussion is based on a fixed point technique for weakly sequentially continuous maps. Next, we apply the abstract result to the weak solvability of a boundary value problem which models the antiplane frictional contact between a cylindrical deformable body and a rigid foundation. In addition, some 3D contact models leading to mixed variational problems with solution-dependent sets of Lagrange multipliers are indicated.*

- Workshop for Young Researchers in Mathematics, May 09-10, 2013 Ovidius University, Constanta, Romania.

  - ***A. Matei**, A unilateral contact model and its weak solvability by a new variational approach. A review of recent results.*

In the present talk we review recent results obtained in the paper [A. Matei, A variational approach via bipotentials for unilateral contact problems, Journal of Mathematical Analysis and Applications, Volume 397, Issue 1, 2013, Pages 371-380]. *A 3D elastostatic frictionless unilateral contact model, for nonlinearly elastic materials was considered. The mechanical model was described mathematically by a boundary value problem consisting of a system of partial differential equations associated with a displacement boundary condition, a traction boundary condition and a contact condition. The contact was modeled by Signorini's contact condition with zero gap neglecting the friction on the potential contact zone. The behavior of the material was expressed by a constitutive law which involves a nonlinear elastic operator, possibly multi-valued. We give a weak formulation using a bipotential function which depends on the constitutive map and its Fenchel conjugate. Thus, we arrive to a system of two variational inequalities whose unknown is the pair consisting of the displacement field and the Cauchy stress field. We prove the existence and the uniqueness of the weak solution based on minimization arguments for appropriate functionals associated with the variational system.*

- XI-eme Colloque Franco-Roumain de Mathematiques Appliquees, Universite de Bucarest, 24-30 Aout 2012, Roumanie.

  - ***A. Matei, M. Sofonea**, Un probleme viscoplastique de contact avec contraintes unilaterales.*

Nous presentons plusieurs conditions aux limites susceptibles de modeliser le contact entre un corps deformable et une fondation. Puis, nous utilisons ces conditions dans la construction d'un modele mathematique decrivant le processus quasistatique de contact pour des materiaux viscoplastiques. Nous etudions ce modele dans Ie cadre de la Theorie Mathematique de la Mecanique du Contact, tout en prouvant l'existence et l'unicite d'une solution faible ainsi que plusieurs resultats de convergence. Les demonstrations sont basees sur des arguments d'inequations quasivariationelles avec terme de memoire. Nous presentons aussi des simulations numeriques qui valident ces resultats de convergence.

- 41-eme Congres National d'Analyse Numerique, SuperBesse- Puy-de-Dome, 21-25 mai 2012, Universite Blaise Pascal, Clermont-Ferrand, France.

  - ***I. Roventa, A. Matei**, On the solvability of an abstract variational system.*

The present talk focuses on the solvability of an abstract variational system which consists of two variational inequalities. The unknown is a pair that we seek into a Cartesian product between two closed, convex, unbounded subsets of two relexive Banach spaces. We prove the existence and the uniqueness of the solution. As we shall see, the unique solution is the unique minimizer of a functional associated to our variational system. Then, the approximation of the solution is discussed by indicating an abstract multilevel algorithm of additive type. Such kind of algorithms were introduced in [L.Badea, Multigrid methods for variational inequalities, Preprint series of the Institute of Mathematics of the Romanian Academy 1, 2010] *for a class*

of abstract variational inequalities. The abstract variational system we investigate is related to the weak solvability of a class of nonlinearly elastic problems. To give an example, we consider a displacement-traction boundary value problem which was recently studied in [*A. C. Matei and C. P. Niculescu, Weak solutions via bipotentials in mechanics of deformable solids, J. Math. Anal. Appl. 379 (2011), No. 1, 15-25*].

- Workshop for Young Researchers in Mathematics, May 10-11, 2012, Ovidius University, Constanta, Romania.

- ***A. Matei****, A quasistatic contact model leading to a history-dependent quasivariational inequality.*

Based on joint work with Mircea Sofonea. The present talk focuses on the weak solvability of a quasistatic contact model formulated on the unbounded interval of time [0;∞). After describing the mechanical model, a weak formulation in a form of a quasivariational inequality involving a history-dependent term is indicated. Based on a fixed point result obtained in [M. Sofonea, C. Avramescu and A. Matei, A Fixed point result with applications in the study of viscoplastic frictionless contact problems, Communications on Pure and Applied Analysis, DOI:10.3934/cpaa.2008.7.645, 7(3), 645-658, 2008], *the existence and the uniqueness of the weak solution is discussed. Several contact models with a similar treatment can be found in* [M. Sofonea and A. Matei, History-dependent quasi-variational inequalities arising in contact mechanics, European Journal of Applied Mathematics, DOI:10.1017/S0956792511000192, vol. 22, 471-491, 2011].

Invited talk

History-dependent operators in Contact Mechanics; October 12, 2012, **Professor Mircea Sofonea**, University of Perpignan, France.

Abstract - Contact phenomena involving deformable bodies arise in industry and everyday life and play important roles in structural and mechanical systems. Owning to the complicated surface physics involved, they lead to new and nonstandard mathematical models. Part of these models are expressed in terms of inequalities governed by history-dependent operators. Such type operators could arise in the constitutive law of the materials or in the frictional contact conditions, as well. In this lecture we present existence and uniqueness results for variational and hemi-variational inequalities with history-dependent operators. Then we use these results in the study of various contact problems involving viscoelastic and viscoplastic materials. In this way we provide the unique weak solvability of the corresponding problems and we complete it with regularity and convergence results. We also present numerical simulations for two-dimensional test problems.

TABEL 1.

Nr.	**ISI articles having the mention of the project PN-II-RU-TE-2011-3-0223**	**Journal**	**Status/ Decision**	**Impact Factor**
1	**A. Matei**, On the solvability of mixed variational problems with solution-dependent sets of Lagrange multipliers; 143(05), October 2013, 1047-1059 http://dx.doi.org/10.1017/S0308210512000637	Proceedings of The Royal Society of Edinburgh, Section: A Mathematics ISSN: 0308-2105.	published	0,777
2	S. Hüeber, **A. Matei**, B. Wohlmuth, A contact problem for electro-elastic materials; DOI: 10.1002/zamm.201200235, 93 (10-11), 789-800, October 2013. Special Issue: Mathematical Modeling: Contact Mechanics, Phase Transitions, Multiscale Problems. In Memory of Christof Eck.	Journal of Applied Mathematics and Mechanics (ZAMM) ISSN: 0044-2267	published	1,008
3	**A. Matei**, A variational approach via bipotentials for unilateral contact problems; Volume 397, Issue 1, 1 January 2013, Pages 371-380. http://dx.doi.org/10.1016/j.jmaa.2012.07.065. WOS: 000309381100031	Journal of Mathematical Analysis and Applications (JMAA) ISSN 0022-247X;	published	1,119
4	I. Andrei, N. Costea and **A. Matei**, Antiplane shear deformation of piezoelectric bodies in contact with a conductive support; DOI: 10.1007/s10898-011-9815-x; Volume 56, Issue 1, pp 103-119, May 2013. WOS: 000317079100006	Journal of Global Optimization **(**JOGO) ISSN: 0925-5001	published	1,355
5	M. Barboteu, **A. Matei** and M. Sofonea, Analysis of Quasistatic Viscoplastic Contact Problems with Normal Compliance;	The Quarterly Journal of Mechanics and Applied	published	0,571

	DOI: 10.1093/qjmam/hbs016 65(4), 555-579, November 2012. WOS: 000310892600005	Mathematics **(**QJMAM) ISSN 0033-5614.		
6	**I. Roventa**, A note on Schur-concave functions; DOI: 10.1186/1029-242X-2012-159 2012:159, 9 pages.	Journal of Inequalities and Applications ISSN: 1029-242X (Springer Open Journal)	published July 2012	0,77
7	**M. Boureanu, A. Matei** and M. Sofonea, Nonlinear problems with p(.)-growth conditions and applications to antiplane contact models. 14 (2) (2014), 295-313, May 2014	Advanced Nonlinear Studies ISSN 1536-1365	published	0,674
8	**I. Roventa**, Generalized equilibrium problems related to Ky Fan inequalities, Abstract and Applied Analysis Volume 2014 (2014**)**, Article ID 301901, 6 pages; http://dx.doi.org/10.1155/2014/301901.	Abstract and Applied Analysis ISSN: 1085-3375 (Print) ISSN: 1687-0409 (Online)	published (online)	1,274
9	M. Barboteu**, A. Matei** and M. Sofonea, On the behavior of the solution of a viscoplastic contact problem. DOI: http://dx.doi.org/10.1090/S0033-569X-2014-01345-4.	Quarterly of Applied Mathematics (QAM) Online ISSN 1552-4485; Print ISSN 0033-569X.	published online September 25 2014	0,536
10	**A. Matei**, A variational approach via bipotentials for a class of frictional contact problems. *DOI:* 10.1007/s10440-014-9868-1	Acta Applicandae Mathematicae (ACTA APPL MATH) ISSN: 0167-8019 (Print) 1572-9036 (Online)	published (online February 7 2014)	0,702
11	**A. Matei**, An existence result for a mixed variational problem arising from Contact Mechanics vol. 20, December 2014, 74-81.	Nonlinear Analysis: Real World Applications (NARWA) ISSN: 1468-1218	published	2,338
12	**A. Matei,** Weak solvability via Lagrange multipliers for contact problems involving multi-contact zones. DOI: 10.1177/1081286514541577.	Mathematics and Mechanics of Solids (MMS)	published (online July 7 2014)	0,860

		Print ISSN: *1081-2865* Online ISSN: *1741-3028*		
13	**A. Matei**, Two abstract mixed variational problems and applications in Contact Mechanics	Nonlinear Analysis: Real World Applications (NARWA) ISSN: 1468-1218	accepted September 2014	2,338

TABEL 2.

Nr.	**Indicator**		**Number of results**
1.	ISI articles	published	12
		accepted	1
2.	BDI published articles		3
3.	Published Research monograph (Cambridge University Press 2012)		1
4.	Submitted articles		2
5.	International conferences		8
6.	Research visits (team members)		6
7.	Invited collaborator (Invited talk)		1

Director,
Lect. dr. Andaluzia-Cristina Matei 04.10.2014