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DEPARTAMENTUL DE MATEMATICA
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Modelare matematica prin ecuatii diferentiale

Modelare matematica prin ecuatii diferentiale
Indrumar pentru laborator

Andaluzia Matei

Cuprins

Laborator 1

Tematica: comenzi si operatii uzuale in Matlab pentru vectori si matrice, prin exemple.

A. Vectori; matrice

A.1

x=[2 3; 4 5; 6 7; 8 9]
r1=size(x,1)
r2=size(x,2)
A=ones(size(x,1),1)
B=zeros(size(x,1),1)
Y=[1 2; 3 4; 5 6; 7 7; 8 8; 9 9]
p=size(Y,1)
G=ones(p,1)

A.2

x=[1 2]
d=size(x,1)
a=ones(size(x,1),1)
b=ones(d,1)

A.3

S=sparse(3,3)
A=ones(3,3)
B=eye(3,3)
S=S+A+B;
b=sparse(3,1);
c=A*b;
d=B*b;
C=zeros(3,3)
D=C+1;
M=[1 0 0; 3 4 0; 2 0 2];
X=sparse(M)

Y=full(M)
N=[1 2 3 4; 5 6 7 8; 9 10 11 12; 13 14 15 16]
Z=sparse(N)
P=[1 0 0 0; 0 2 3 0; 0 0 0 3; 9 0 0 0]
P=sparse(P)
P=sparse(4,4)
N=N+P

A.4

x=[2 3; 4 5; 6 7; 8 9]
r1=size(x,1)
r2=size(x,2)
A=ones(size(x,1),1)
B=zeros(size(x,1),1)
Y=[1 2; 3 4; 5 6; 7 7; 8 8; 9 9]
p=size(Y,1)
G=ones(p,1)

A.5

A=[1 0 5; 2 1 6; 3 2 6]
B=[1 0; 2 3; 4 5; 6 7; 8 9; 2 3]
C=A(2,:)
D=B(C,:)
sum(D)
sum(A)
x=[1 2 3 4 5 6 7 8]
sum(x)
b=sparse(size(B,1),1)
b=[4 4 9 9 9 4]
Z=b(C)

A.6

A=[1 0; 0 1];
B=[1 1; 1 1];
C=B*A;
E=A.*B
D=inv(A);

A.7

v=[3 4]

norm(v)

A.8

prod(1:2)
prod(1:3)
prod(1:4)
prod(1:2:8)
prod(1:3:12)

A.9

v=[1 2 3 5 6 7]
a=unique(v)
A=[1 2; 3 4; 1 2]
b=unique(A)
w=[1 1 2 2 3 3]
c=unique(w)

A.9

A=[1 2; 3 4; 5 6; 6 7; 8 9]
v=[3 4 5]
w=setdiff(1:size(A,1),v)
v2=[1 2 5]
w2=setdiff(1:size(A,1),v2)
v3=1
w3=setdiff(1:size(A,2),v3)
size(A,1)
size(A,2)

Laborator 2

Tematica: Comenzi si operatii uzuale in MATLAB (II)(curbe, suprafete) **B.Trasare curbe**

B:1

```
x=0.05:0.5:30;
subplot(2,1,1);
y=sin(x)+x-x.*cos(x)
plot(x,y);
xlabel('x');
ylabel('y');
title('primul desen')
x=-100:0.5:100;
subplot(2,1,2);
y=sin(x)+x-x.*cos(x)
plot(x,y,'r');
xlabel('x');
ylabel('y');
title('al doilea desen')
```

B:2

```
x=0:10;
y=2.^x;
plot(x,y,'r–')
grid
```

B:3

```
x=0:.01:4;
y=50*exp(log(12/5)*x);
subplot(2,1,1);
plot(x,y,'r')
N=20:.01:200
z=log(N)-log(201-N)-log(20)+log(181);
```

z=z/50;
subplot(2,1,2);
plot(z,N,'g')
B:4
x=1:.1:4;
y1=sin(3*x);
y2=cos(5*x);
y3=sin(3*x).*cos(5*x);
subplot(1,3,1); plot(x,y1); title('1')
legend('desen 1');
subplot(1,3,2); plot(x,y2); title('2')
legend('desen 2');
subplot(1,3,3); plot(x,y3); title('3')
legend('desen 3');
B:5
subplot(1,3,1)
x=-2:.0001:2;
y=abs(x);
plot(x,y,'r')
title('(a)')
subplot(1,3,2)
x=-2:.0001:2;
`y=sqrt(x.^2+0.25)-0.5`;
plot(x,y,'g')
title('(b)')
subplot(1,3,3)
x=-2:.0001:2;
`y=abs(x).^1.5./1.5`;
plot(x,y,'b')
title('(c)')
B:6
r=1;
t=0:0.005:2*pi;
x=r.*cos(t);
y=r.*sin(t);
plot(x,y,'r')

B:7

```
x=0:.01:4;
y=50*exp(log(12/5)*x);
plot(x,y,'r')
```

C. Suprafete

C:1

```
[X,Y]=meshgrid(-1:.05:1, -1:.05:2);
Z=sin(5*X).*cos(2*Y);
mesh(X,Y,Z);
xlabel('x')
ylabel('y')
title('title')
```

C:2

```
x=0:.2:2;
y=0:.2:2;
[X Y]=meshgrid(x,y);
Z=Y.*exp(-(X.^2+Y.^2));
surf(X,Y,Z)
xlabel('x')
ylabel('y')
title('mesh')
```

C:3

```
x=-2:.2:2;
y=-2:.2:2;
[X Y]=meshgrid(x,y);
Z=Y.*exp(-(X.^2+Y.^2));
mesh(X,Y,Z)
xlabel('x')
ylabel('y')
title('mesh')
```

C:4

```
x=-2:.5:2;
y=-2:.5:2;
[X Y]=meshgrid(x,y);
Z=sin(X)+cos(Y);
```

```
subplot(1,3,1)
surf(X,Y,Z)
xlabel('x')
ylabel('y')
title('s1')
x=-2:.5:2;
y=-2:.5:2;
[X Y]=meshgrid(x,y);
Z=sin(X).*cos(Y);
subplot(1,3,2)
surf(X,Y,Z)
xlabel('x')
ylabel('y')
title('s1')
x=-2:.5:2;
y=-2:.5:2;
[X Y]=meshgrid(x,y);
Z=X.^2+Y.^4;
subplot(1,3,3)
surf(X,Y,Z)
xlabel('x')
ylabel('y')
title('s1')
```

Laborator 3

Tematica: modelarea miscarii punctului material pe orizontala

- Pozitie bila -sol exacta (frecare neglijabila)

```
t=0:.5:9;
x0=0;
v0=7;
x=v0.*t+x0;
plot(t,x,'r')
```

- Pozitie bila-sol. aprox

```
[teul,yeul]=eul('funct2',[0 9],0,.5)
plot(teul,yeul,'c')
```

- Pozitie bila-sol.exacta- frecare neneglijabila

```
t=0:.5:9;
m=5;
k=6;
v0=4;
x0=5;
x=x0+v0.*m/k-v0.*m.*exp(-k.*t/m)/k;
plot(t,x,'r')
```

- Pozitie-viteza (sol. aprox)

```
[teul,yeul]=eul('funct',[0 9],[0 7],.5)
subplot(1,2,1)
plot(teul,yeul (:,1),'g')
subplot(1,2,2)
plot(teul,yeul(:,2),'m')
```

- Viteza bilei-frecare neglijabila

```
t=0:.5:9;
a=size(t)
b=a(2)
v0=5;
A=ones(b,1)
v=v0.*A;
plot(t,v)
```

- Viteza bilei-frecare neneglijabila-sol. aprox

```
[teul,yeul]=eul('funct3',[0 9],7,.5)
plot(teul,yeul,'c')
```

- Viteza bilei-sol exacta-frecare neneglijabila

```
t=0:.5:9;
m=5;
k=6;
v0=4;
v=v0.*exp(-k.*t/m);
plot(t,v,'r')
```

- Pozitia bilei in cazul in care se neglijeaza frecare- studiul erorii

```
t=0:.5:9;
v0=7;
x0=0;
x=v0.*t+x0;
[teul,yeul]=eul('funct2',[0 9],0,.5)
plot(t,x,'r')
hold on
plot(teul,yeul,'g')
```

S-au utilizat urmatoarele functii:

```
funct.m
function w=funct(t,y)
w(1)=y(2);
w(2)=0
```

funct2.m

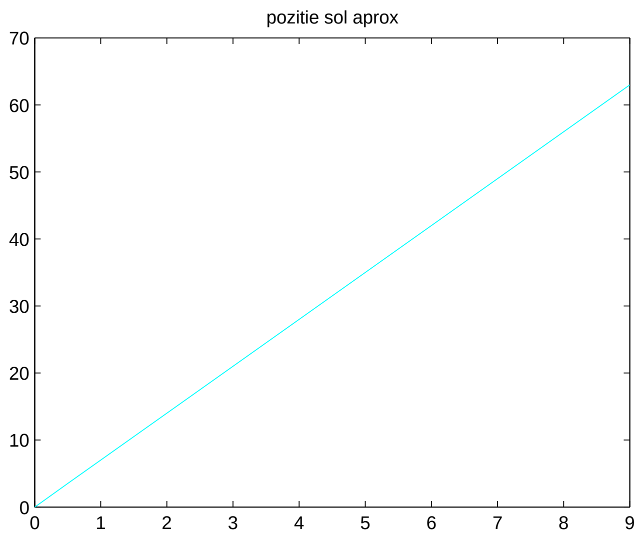

Figura 1

```
function w=funct2(t,y)
v0=7;
w=v0;
```

```
funct3.m
function w=funct3(t,y)
k=2;
m=5;
w=-k*y/m;
```

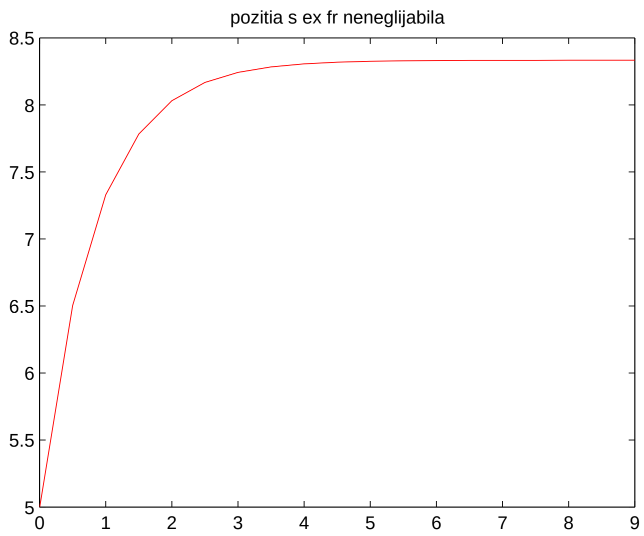

FIGURA 2

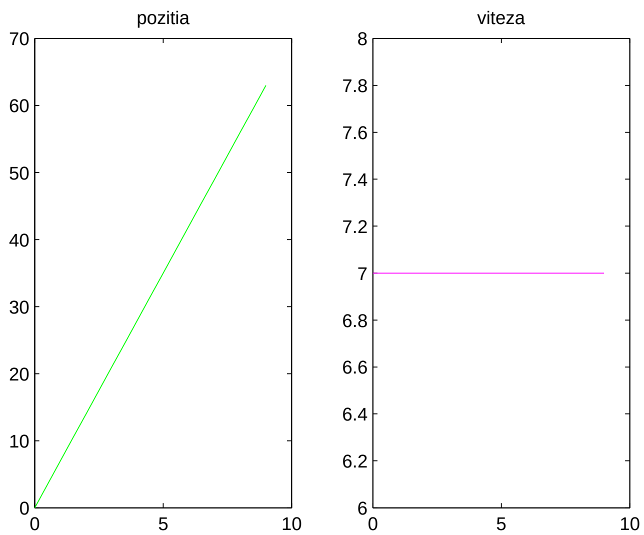

FIGURA 3

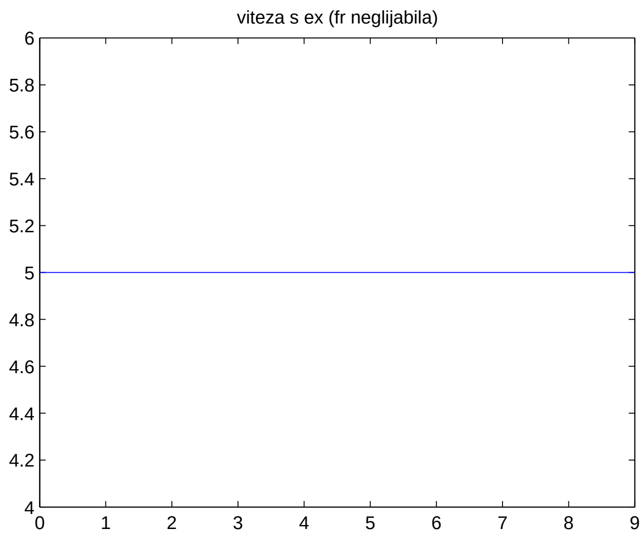

FIGURA 4

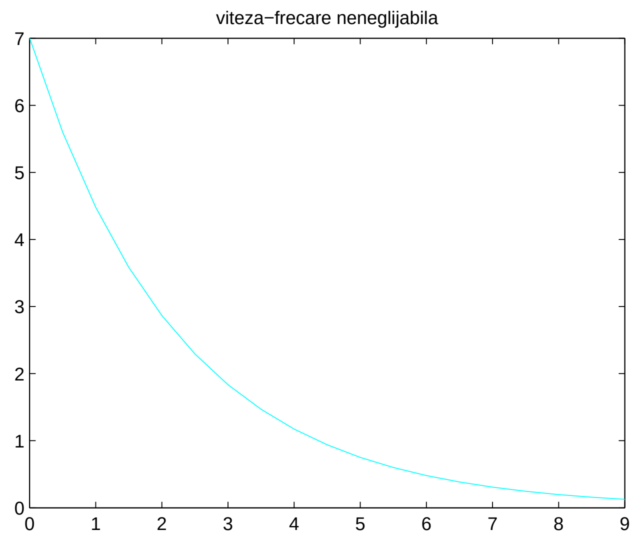

FIGURA 5

Laborator 4

Tematica: modelarea miscarii punctului material pe verticala (cadere libera, cadere in prezenta fortei de frecare)

- Cadere libera, pozitia si viteza bilei-solutii exacte

```
t=0:.25:9;
g=10;
v=g.*t;
x0=3;
x=g.*t.^2/2+x0;
subplot(1,2,1)
plot(t,x)
subplot(1,2,2)
plot(t,v,'r')
```

- Cadere libera, pozitia si viteza bilei-solutii aproximative

```
[ta,yeul]=eul('functcl',[0 9],[3 0],.25)
subplot(1,2,1)
plot(ta,yeul(:,1),'g')
subplot(1,2,2)
plot(ta,yeul(:,2),'m')
```

- Pozitia si viteza bilei in cadere libera (forta de frecare neglijabila), solutia exacta si solutii aproximative t=0:.5:9;

```
x0=3;
g=10;
v=g.*t;
x=g.*t.^2/2+x0;
[ta,y]=eul('functcl',[0 9],[3 0],.5);
[ta,y2]=rk2('functcl',[0 9],[3 0],.5);
[ta,y4]=rk4('functcl',[0 9],[3 0],.5);
```

```
subplot(1,2,1)
plot(t,x,'r')
hold on
plot(ta,y(:,1),'g')
plot(ta,y2(:,1),'m')
plot(ta,y4(:,1),'c')
subplot(1,2,2)
plot(t,v,'r')
hold on
plot(ta,y(:,2),'g')
plot(ta,y2(:,2),'m')
plot(ta,y4(:,2),'c')
```

- Viteza bilei in cadere pe verticala cu forta de frecare neneglijabila, solutie exacta si solutie aproximativa

```
t=0:.5:9;
m=3;
g=10;
k=5;
A=m*g/k;
B=exp(-k*t/m);
v=A-A*B;
[teul,yeul]=eul('funct',[0 9],0,.5)
plot(t,v,'r')
hold on
plot(teul,yeul,'g')
```

S-au utilizat urmatoarele functii:

funct.m

```
function w=funct(t,y)
g=10;
k=5;
m=3;
w=g-k*y/m;
```

functcl.m

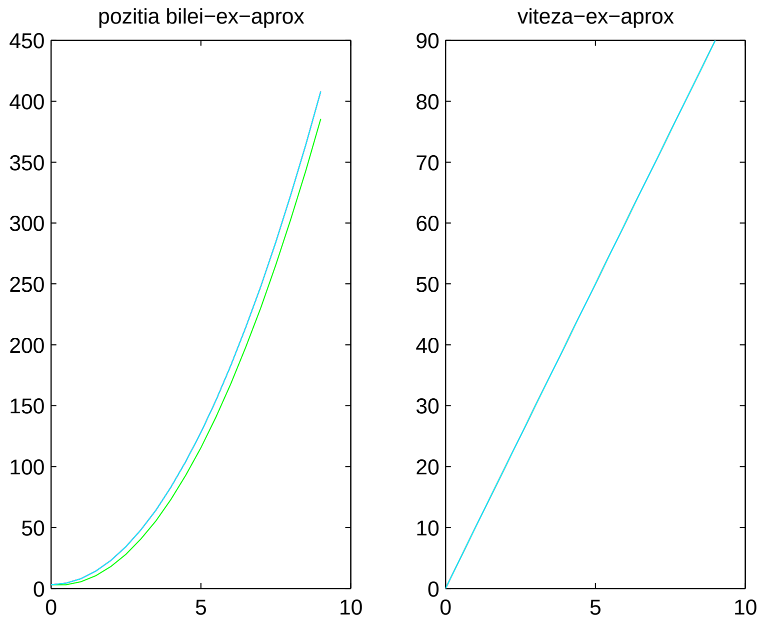

FIGURA 6

```
function w=functcl(t,y)
g=10;
w(1)=y(2);
w(2)=g;
```

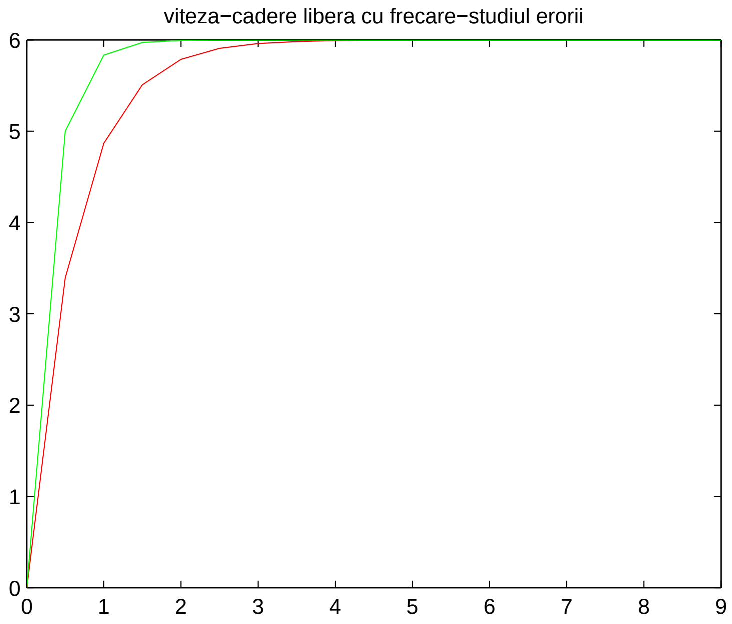
viteza−cadere libera cu frecare−studiul erorii
0
1
2
3
4
5
6
0
1
2
3
4
5
6
7
8
9

Figura 7

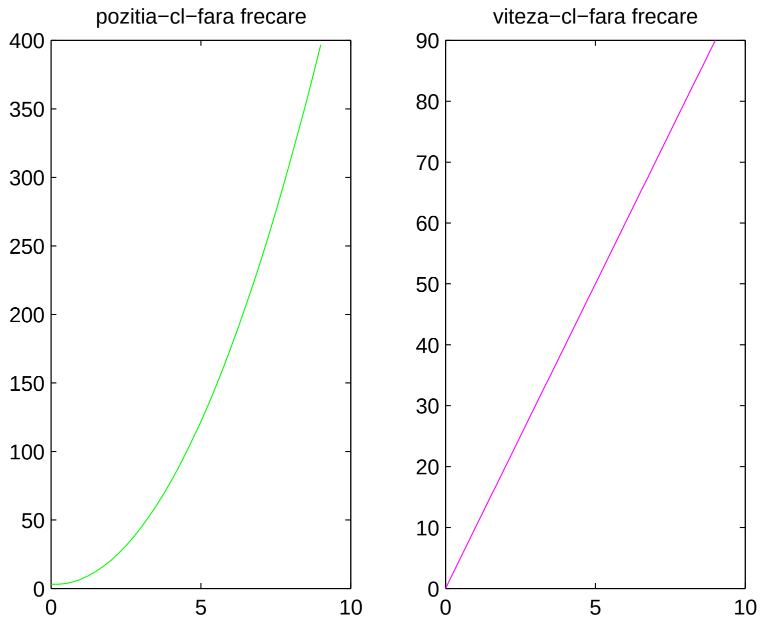
pozitia−cl−fara frecare
400
350
300
250
200
150
100
50
0
0
5
10
viteza−cl−fara frecare
90
80
70
60
50
40
30
20
10
0
0
5
10

FIGURA 8

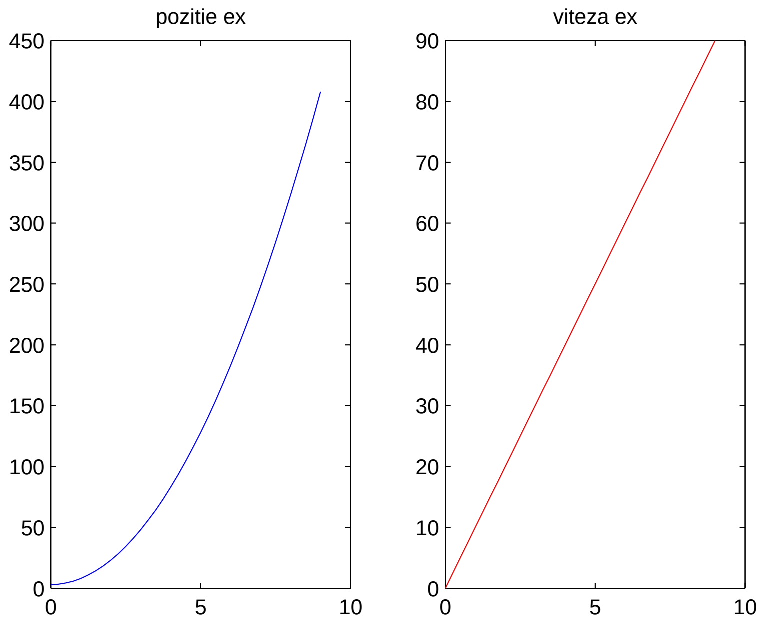
pozitie ex
450
400
350
300
250
200
150
100
50
0
0
5
10
viteza ex
90
80
70
60
50
40
30
20
10
0
0
5
10

FIGURA 9

Laborator 5

Tematica: modelarea miscarii sistemelor masa-resort, masa-resort-piston; oscilatii libere/amortizate/perturbate

- Modelarea miscarii unui sistem masa -resort (oscilatii libere)
```
t=0:.5:9;
[teul,yeul]=eul('funct',[0 9],[3 7],.25)
plot(teul,yeul(:,1),'r')
```
- Comparare solutii aproximative pentru oscilatii libere
```
[t,yeul]=eul('funct',[0 9],[3 7],.25)
[t,yrk2]=rk2('funct',[0 9],[3 7],.25)
[t,yrk4]=rk4('funct',[0 9],[3 7],.25)
plot(t,yeul(:,1),'r')
hold on
plot(t,yrk2(:,1),'k')
plot(t,yrk4(:,1),'go')
```
- Pozitie si viteza pentru un punct material intr-un sistem masa-resort
```
[t,yeul]=eul('funct',[0 9],[3 7],.25)
subplot(2,1,1)
plot(t,yeul(:,1),'r')
subplot(2,1,2)
plot(t,yeul(:,2),'g')
```
- Modelarea miscarii unui sistem masa-resort-piston (oscilatii amortizate)
```
[teul,yeul]=eul('funct2',[0 50],[3 7],.25)
plot(teul,yeul(:,1),'r')
```
- Pozitie si viteza in oscilatii amortizate
```
[teul,yeul]=eul('funct2',[0 9],[3 7],.25)
subplot(2,1,1)
plot(teul,yeul(:,1),'r')
subplot(2,1,2)
plot(teul,yeul(:,2),'g')
```

- Oscilatii perturbate

```
[t,y]=rk4('oscp',[0,4],[5,0],.1);
plot(t,y(:,1),'r')
```

- Oscilatii libere; oscilatii amortizate; oscilatii perturbate

```
[t,y1]=rk4('funct',[0,50],[5,0],.1);
[t,y2]=rk4('funct2',[0,50],[5,0],.1);
[t,y3]=rk4('oscp',[0,50],[5,0],.1);
subplot(3,1,1)
plot(t,y1(:,1),'r')
subplot(3,1,2)
plot(t,y2(:,1),'r')
subplot(3,1,3)
plot(t,y3(:,1),'r')
```

S-au utilizat urmatoarele functii

funct.m

```
function w=funct(t,y)
k=3;
m=10;
w(1)=y(2);
w(2)=-k*y(1)/m;
```

funct2.m

```
function w=funct2(t,y)
k=3;
m=10;
k1=5;
w(1)=y(2);
w(2)=-k1*y(2)/m-k*y(1)/m;
```

oscp.m

```
function w=oscp(t,Y)
k=3;
m=10;
k1=5;
w(1)=Y(2);
```

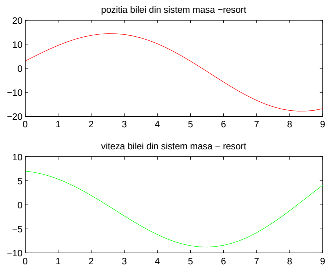

FIGURA 10

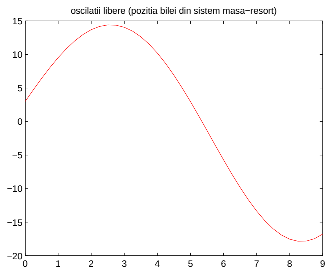

FIGURA 11

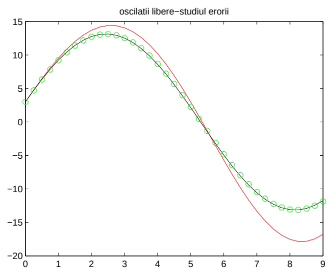

FIGURA 12

w(2)=-k1*Y(2)/m-k*Y(1)/m+10*cos(7*t)

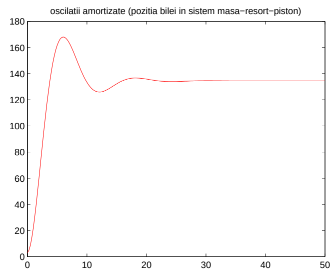

FIGURA 13

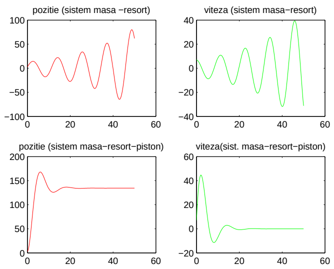

FIGURA 14

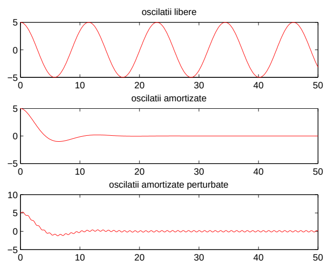

FIGURA 15

Laborator 6

Tematica: dinamica populatiilor I (modelul exponential, modelul logistic)

- Model exponential-solutia exacta

```
t=0:.5:9;
P0=20;
k=.3;
P=P0.*exp(k.*t);
plot(t,P,'r')
```

- Model exponential solutie aprox [teul,yeul]=eul('funct',[0 9], 20, .5);

```
plot(teul,yeul,'c')
```

- Model exponential-studiul erorii in aproximarea solutiei

```
t=0:.2:9;
P0=20;
k=.3;
P=P0.*exp(k.*t);
[ta,yeul]=eul('funct',[0 9],20,.2);
[ta,yrk2]=rk2('funct',[0 9],20,.2);
[ta,yrk4]=rk4('funct',[0 9],20,.2);
plot(t,P,'r')
hold on
plot(ta,yeul,'c')
plot(ta,yrk2,'g')
plot(ta,yrk4,'*')
```

- Modelul logistic -solutia exacta

```
t=0:.2:50;
P0=20;
k=.3;
M=30;
```

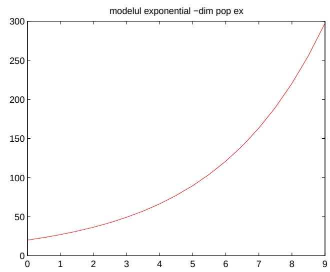

FIGURA 16

P=P0*M./((M-P0).*exp(-k.*t)+P0);
plot(t,P,'r')

- Modelul logistic -solutia aproximativa [teul,yeul]=eul('functl',[0 50], 20, .2);
  plot(teul,yeul,'c')

S-au utilizat urmatoarele functii

funct.m
function w=funct(t,y)
k=.3;
w=k*y;

functl.m
function w=functl(t,y)
k=.3;
M=30;
w=k*y*(1-y/M);

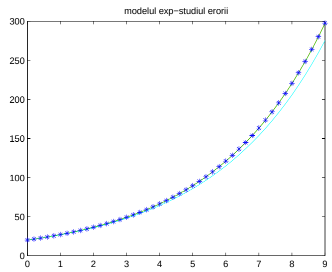

FIGURA 17

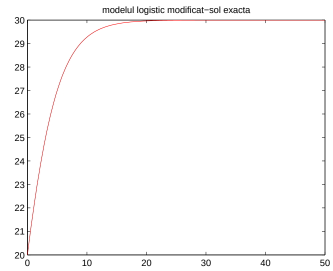

FIGURA 18

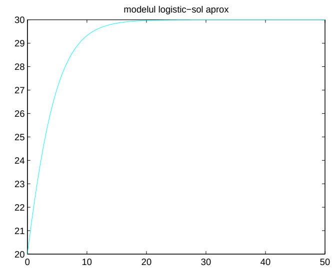

FIGURA 19

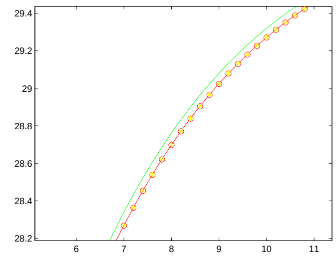

FIGURA 20

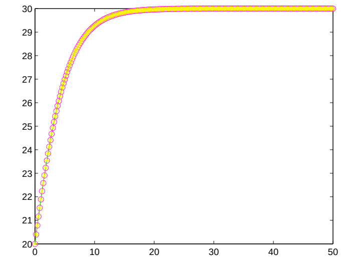

FIGURA 21

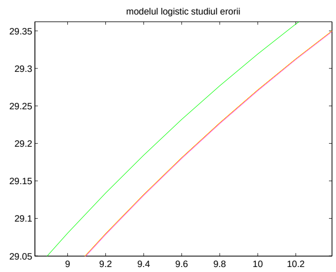

FIGURA 22

Laborator 7

Tematica: dinamica populatiilor II (modelul logistic modificat)

- Modelul logistic modificat [t,y]=rk4('funct2',[0 50],20,.2);
  ```
  plot(t,y,'r')
  ```

- Modelul logistic modificat- un studiu al erorii bazat pe trei metode de aproximare
  ```
  [t,y1]=rk4('funct2',[0 10],20,.2);
  [t,y2]=rk2('funct2',[0 10],20,.2);
  [t,y3]=eul('funct2',[0 10],20,.2);
  plot(t,y3,'r')
  hold on
  plot(t,y2,'g')
  plot(t,y1)
  ```

- Modelul logistic modificat versus modelul logistic
  ```
  [t,ylog]=rk4('functl',[0 50], 20, .2);
  [t,ylogmod]=rk4('funct2',[0 50],20,.2);
  plot(teul,ylog,'g')
  hold on
  plot(t,ylogmod,'r')
  ```

- model exponential/model logistic/ model logistic modificat
  ```
  [teul,yeul]=eul('funct2bis',[0 10],10,.2);
  plot(teul,yeul,'r')
  hold on
  P0=10;
  M=30;
  k=.3;
  Pe=P0.*exp(k.*teul)
  plot(teul,Pe,'co')
  ```

```
Pl=P0*M./((M-P0).*exp(-k.*teul)+P0);
plot(teul,Pl,'g')
```

S-au utilizat urmatoarele functii

```
funct2.m
function w=funct2(t,y)
function w=funct2(t,y)
k=.3;
M=30;
N=7;
w=k*y*(1-y/M)*(y/N-1);
```

```
functl.m
function w=functl(t,y)
k=.3;
M=30;
w=k*y*(1-y/M);
```

```
funct.m
function w=funct(t,y)
k=.3;
w=k*y;
```

```
funct2bis.m
function w=funct2bis(t,y)
k=.3;
M=30;
N=2;
w=k*y*(1-y/M)*(y/N-1);
```

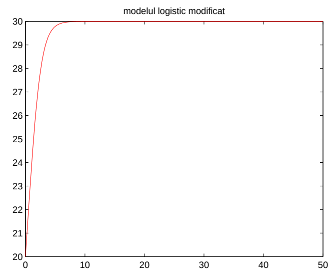

FIGURA 23

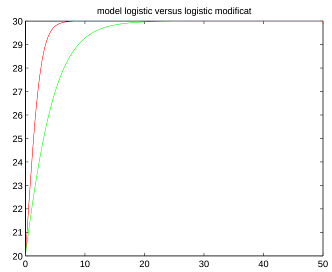

FIGURA 24

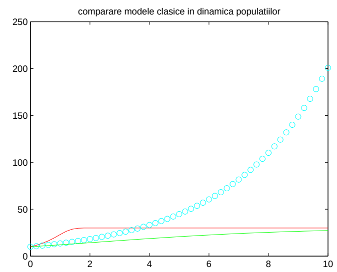

FIGURA 25

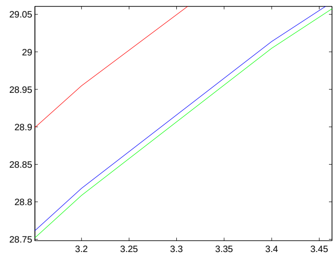

FIGURA 26

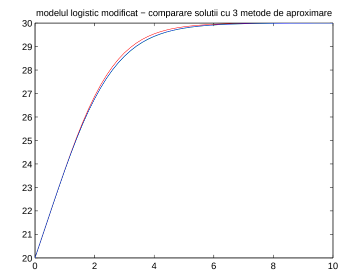

FIGURA 27

Laborator 8

Tematica: dinamica populatiilor III(interactiunea dintre specii: prada-pradator; prada-pradator-competitor)

- Interactiunea dintre doua specii (Model prada-pradator: exemplul 1)

```
[teul,yeul]=eul('funct',[0 18],[5 8],.25);
plot(teul,yeul(:,1),'g')
hold on
plot(teul,yeul(:,2),'r')
```

- Interactiunea dintre doua specii (Model prada-pradator: exemplul 2)

```
[t,y]=rk4('pp',[0 12],[10 100],1);
plot(t,y(:,1),'r')
hold on
plot(t,y(:,2),'g')
```

- Interactiunea dintre doua specii (Model prada-pradator: exemplul 3)

```
[t,y]=rk4('pp',[0 12],[100 10],1);
plot(t,y(:,1),'r')
hold on
plot(t,y(:,2),'g')
```

- Interactiunea dintre trei specii (Model prada-pradator-competitor)

```
[teul,yeul]=eul('funct1',[0 9 17],[5 8 10],.25)
plot(teul,yeul(:,1),'g')
hold on
plot(teul,yeul(:,2))
plot(teul,yeul(:,3),'r')
```

- Dependenta de parametri

```
[teul,yeul1]=eul('funct3',[0 18],[5 8],.25)
[teul,yeul2]=eul('funct4',[0 18],[5 8],.25)
[teul,yeul3]=eul('funct5',[0 18],[5 8],.25)
```

```
subplot(2,1,1)
plot(teul,yeul1(:,1),'g')
hold on
plot(teul,yeul2(:,1),'c')
plot(teul,yeul3(:,1),'r')
subplot(2,1,2)
plot(teul,yeul1(:,2),'r')
hold on
plot(teul,yeul2(:,2),'c')
plot(teul,yeul3(:,2),'g')
```

- Dependenta de data initiala (exemplul 1)

```
[teul,yeul1]=eul('funct',[0 9],[12 18],.25)
[teul,yeul2]=eul('funct',[0 9],[13 17],.25)
subplot(2,1,1)
plot(teul,yeul1(:,1),'g')
hold on
plot(teul,yeul2(:,1),'r')
subplot(2,1,2)
plot(teul,yeul1(:,2),'c')
hold on
plot(teul,yeul2(:,2),'g')
```

- Dependenta de data initiala (exemplul 2)

```
[teul,yeul1]=eul('funct',[0 9],[12 18],.25)
[teul,yeul2]=eul('funct',[0 9],[11 19],.25)
subplot(2,1,1)
plot(teul,yeul1(:,1),'g')
hold on
plot(teul,yeul2(:,1),'r')
subplot(2,1,2)
plot(teul,yeul1(:,2),'c')
hold on
plot(teul,yeul2(:,2),'g')
```

- Dependenta de data initiala (exemplul 3)

```
[teul,yeul1]=eul('funct1',[0 9 17],[5 8 10],.25)
```

```
[teul,yeul2]=eul('funct1',[0 9 17],[3 5 7],.25)
[teul,yeul3]=eul('funct1',[0 9 17],[2 4 6],.25)
subplot(2,1,1)
plot(teul,yeul1(:,1),'g')
hold on
plot(teul,yeul2(:,1),'r')
plot(teul,yeul3(:,1),'co')
subplot(2,1,2)
plot(teul,yeul1(:,2),'c')
hold on
plot(teul,yeul2(:,2),'g')
plot(teul,yeul3(:,2),'r*')
```

- Dependenta de data initiala (exemplul 4)

```
[teul,yeul1]=eul('funct',[0 9],[12 18],.25)
[teul,yeul2]=eul('funct',[0 9],[7 10],.25)
subplot(2,1,1)
plot(teul,yeul1(:,1),'g')
hold on
plot(teul,yeul2(:,1),'r')
subplot(2,1,2)
plot(teul,yeul1(:,2),'c')
hold on
plot(teul,yeul2(:,2),'g')
```

S-au utilizat urmatoarele functii:

```
funct.m
function w=funct(t,y)
a=.2; b=.03; c=.3; k=.01;
w(1)=a*y(1)-b*y(1)*y(2);
w(2)=-c*y(2)+k*y(1)*y(2)
```

```
funct1.m
function w=funct1(t,y)
a1=2; a2=2; a3=2.5; a11=.3; a12=.2; a13=.2; a22=.3; a21=.2;
a23=.2; a33=.3; a31=.2; a32=.3;
```

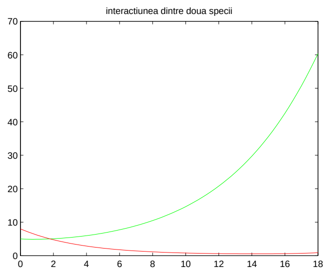

Figura 28

```
w(1)=a1*y(1)-a11*y(1)^2-a12*y(1)*y(2)-a13*y(1)*y(3);
w(2)=a2*y(2)-a22*y(2)^2-a21*y(1)*y(2)-a23*y(3)*y(2);
w(3)=a3*y(3)-a33*y(3)^2-a31*y(1)*y(2)-a32*y(2)*y(3);
```

```
funct3.m
function w=funct3(t,y)
a1=.4; b=.03; c=.3; k=.01;
w(1)=a1*y(1)-b*y(1)*y(2);
w(2)=-c*y(2)+k*y(1)*y(2)
```

```
funct4.m
function w=funct4(t,y)
a2=.9; b=.03; c=.3; k=.01;
w(1)=a2*y(1)-b*y(1)*y(2);
w(2)=-c*y(2)+k*y(1)*y(2)
```

```
funct5.m
function w=funct5(t,y)
a3=.27; b=.03; c=.3; k=.01;
w(1)=a3*y(1)-b*y(1)*y(2);
w(2)=-c*y(2)+k*y(1)*y(2)
```

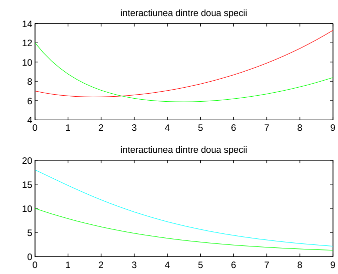

Figura 29

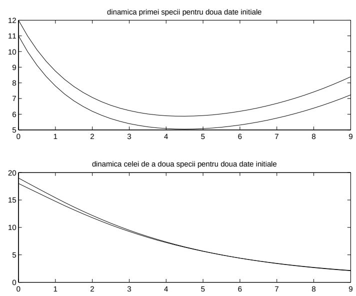

Figura 30

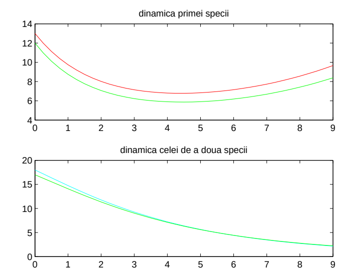

Figura 31

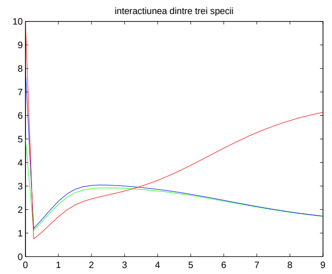

FIGURA 32

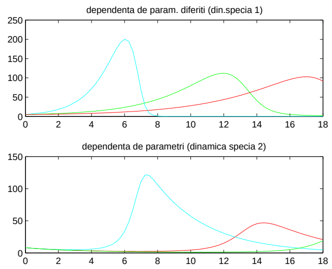

FIGURA 33

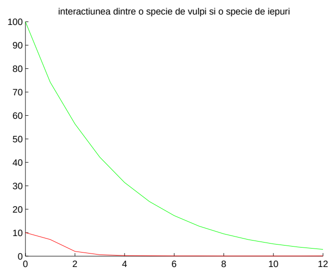

FIGURA 34

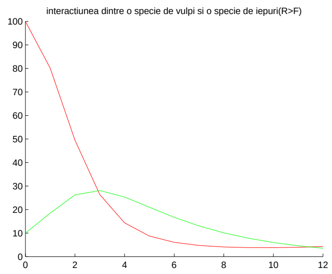
interactiunea dintre o specie de vulpi si o specie de iepuri(R>F)
100
90
80
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12

FIGURA 35

Laborator 9

Tematica: modelare cu aplicatii in medicina I

- Modelare dinamica HIV (model 1)

```
t=0:.5:10;
N0=26;
a=.2;
N=N0*exp(-a*t);
plot(t,N,'r')
b=log(2)/a;
```

- Modelare dinamica HIV (model 2)

```
t=0:.5:10;
N0=26;
a0=.2;
N=N0*exp(-a0*t.^2/2);
plot(t,N,'r')
b=sqrt(2*log(2)/a0);
```

- Modelare dinamica HIV (model 3)

```
t=0:.5:10;
N0=26;
a0=.2;
c=3;
N=N0*exp(-a0*(t.^(c+1))/c+1);
plot(t,N,'c')
b=((c+1)*log(2)/a0).^1/(c+1);
```

- Model 2 -sol aprox

```
[teul,yeul]=eul('funct1',[0 10],26,.5)
plot(teul,yeul,'r')
b=sqrt(2*log(2)/a0);
```

- Model 3-sol.aprox

  [teul,yeul]=eul('funct2',[0 10],26,.5)
  plot(teul,yeul,'r')
  b=((c+1)*log(2)/a0).^1/(c+1);

- Comparare-model 2si model 3

  [t,y]=eul('funct2',[0 10],26,.5)
  [t,y1]=eul('funct3',[0 10],26,.5)
  plot(t,y,'r')
  hold on
  plot(t,y1,'g')

- Comparare solutii aprox

  [teul,yeul1]=eul('funct',[0 10],26,.5)
  [teul,yeul2]=eul('funct1',[0 10],26,.5)
  [teul,yeul3]=eul('funct2',[0 10],26,.5)
  [teul,yeul4]=eul('funct3',[0 10],26,.5)
  subplot(1,4,1)
  plot(teul,yeul1,'r')
  subplot(1,4,2)
  plot(teul,yeul2,'c')
  subplot(1,4,3)
  plot(teul,yeul3,'*')
  subplot(1,4,4)
  plot(teul,yeul4,'g')

- Model 1 -sol aprox

  [teul,yeul]=eul('funct',[0 10],26,.5)
  plot(teul,yeul,'g')

S-au utilizat urmatoarele functii:

```
funct.m
function w=funct(t,y)
a=.2;
w=-a*y;
```

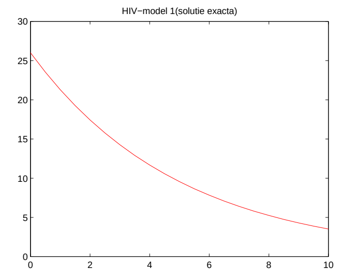

FIGURA 36

```
funct1.m
function w=funct1(t,y)
a0=.2;
w=-a0*t*y;

funct2.m
function w=funct2(t,y)
a0=.2;
c=1.2;
w=-a0*t^c*y;

funct3.m
function w=funct3(t,y)
a0=.2;
c=1.4;
w=-a0*t^c*y;
```

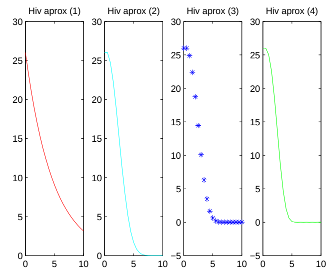

FIGURA 37

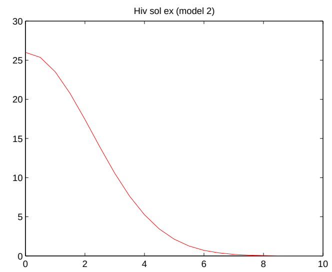

FIGURA 38

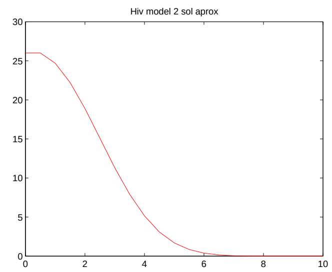

FIGURA 39

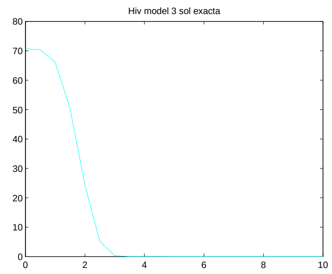

FIGURA 40

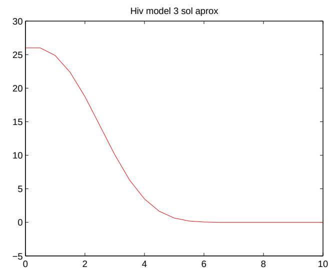

FIGURA 41

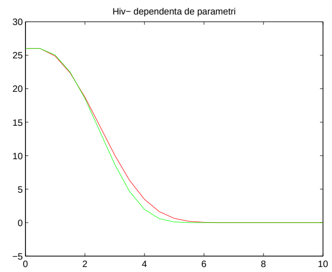

FIGURA 42

Figura 43

Laborator 10

Tematica: modelare in medicina II (modelul Gompertz)

- Model Gompertz-sol. aprox

```
[t,y]=eul('funct4',[0 9], 20,.05)
plot(t,y,'c')
```

- Model Gompertz generalizat -sol. aprox

```
clear all
[t,y]=eul('func6',[0 9], 20,.05)
plot(t,y)
```

- Comparare Gompertz versus Gompertz generalizat cu doua valori pentru a

```
[t,y]=eul('funct4',[0 9], 20,.05);
[t,y1]=eul('funct5',[0 9], 20,.05);
[t,y2]=eul('func6',[0 9], 20,.05);
subplot (1,2,1)
plot(t,y,'r')
hold on
plot(t,y1,'g')
subplot(1,2,2)
plot(t,y,'r')
hold on
plot(t,y2)
```

- Dependenta de parametri

```
clear all
[t,y1]=eul('funct106',[0 9], 20,.05)
[t,y4]=eul('funct109',[0 9], 20,.05)
[t,y5]=eul('funct5',[0 9], 20,.05)
plot(t,y1,'r')
hold on
```

```
plot(t,y4)
plot(t,y5,'g')
```

- Efectele alegerii pasului de discretizare!!!!

```
clear all
[t,y1]=eul('funct5',[0 9], 20,.5)
[t2,y2]=eul('funct5',[0 9], 20,.2)
plot(t,y1,'r')
hold on
plot(t2,y2,'g')
```

s-au utilizat urmatoarele functii:

```
func6.m
function w=func6(t,y)
r=2;
k=7;
a=.5;
w=r/a*y*(1-(y/k)^a);
```

```
funct5.m
function w=funct5(t,y)
r=2;
k=7;
a=.00000089;
w=r/a*y*(1-(y/k)^a);
```

```
funct4.m
function w=funct4(t,y)
r=2;
k=7;
w=-r*y*log(y/k);
```

```
funct106.m
function w=funct106(t,y)
r=2;
k=7;
```

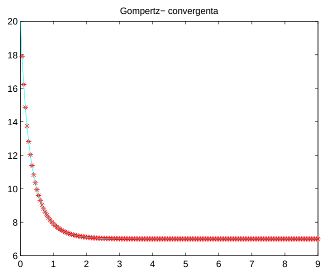

FIGURA 44

```
a=1;
w=r/a*y*(1-(y/k)^a);
```

funct107.m

```
function w=funct107(t,y)
r=2;
k=7;
a=.2;
w=r/a*y*(1-(y/k)^a);
```

funct108.m

```
function w=funct108(t,y)
r=2;
k=7;
a=3;
w=r/a*y*(1-(y/k)^a);
```

funct109.m

```
function w=funct109(t,y)
r=2;
k=7;
a=.5;
w=r/a*y*(1-(y/k)^a);
```

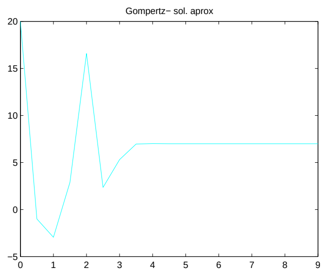

FIGURA 45

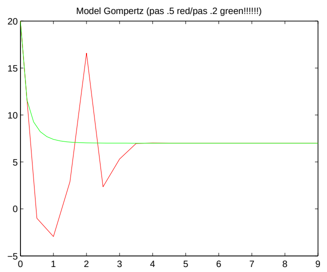

FIGURA 46

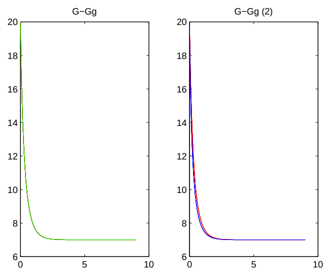

FIGURA 47

Laborator 11

Tematica: modelare in medicina III (modele de dezvoltare tumora)

- Model dezvoltare naturala tumora-sol. exacta

```
t=0:.25:9;
N0=4;
a=1.5;
a1=2;
N=exp((a1-a)*t)*N0;
plot(t,N,'c')
```

- Model dezvoltare naturala tumora-sol. aprox

```
[teul,yeul]=eul('funct1',[0 9], 4,.25);
[trk2,yrk2]=rk2('funct1',[0 9], 4,.25);
[trk4,yrk4]=rk4('funct1',[0 9], 4,.25);
plot(teul,yeul,'go')
hold on
plot(trk2,yrk2,'c*')
plot(trk4,yrk4,'r')
```

- Comparare sol.exacta cu sol. aprox

```
t=0:.25:9;
N0=4;
a=1.5;
a1=2;
N=exp((a1-a)*t)*N0;
[teul,yeul]=eul('funct1',[0 9], 4,.25);
[trk2,yrk2]=rk2('funct1',[0 9], 4,.25);
[trk4,yrk4]=rk4('funct1',[0 9], 4,.25);
plot(t,N,'o')
hold on
```

```
plot(teul,yeul,'g')
plot(trk2,yrk2,'*')
plot(trk4,yrk4,'r')
```

- Model dezv tumora II t=0:.25:50;

```
N0=5;
D=20;
N=exp(log(2)/D*t)*N0;
plot(t,N,'c')
```

- Model II-sol. aprox

```
[teul,yeul]=eul('funct',[0 50], 5,.25);
[trk2,yrk2]=rk2('funct',[0 50], 5,.25);
[trk4,yrk4]=rk4('funct',[0 50], 5,.25);
plot(teul,yeul,'go')
hold on
plot(trk2,yrk2,'c*')
plot(trk4,yrk4,'r')
```

- Model II-comparare

```
t=0:.25:50;
N0=5;
D=20;
N=exp(log(2)/D*t)*N0;
[teul,yeul]=eul('funct',[0 50], 5,.25);
[trk2,yrk2]=rk2('funct',[0 50], 5,.25);
[trk4,yrk4]=rk4('funct',[0 50], 5,.25);
plot(t,N,'mo')
hold on
plot(teul,yeul,'g')
plot(trk2,yrk2,'c*')
plot(trk4,yrk4,'r')
```

- Model 1 versus model 2

```
clear all;
t=0:.25:10;
N0=5;
```

```
D=2;
N=exp(log(2)/D*t)*N0;
plot(t,N,'r')
hold on
a=1.5;
a1=2;
N=exp((a1-a)*t)*N0;
plot(t,N,'g')
```

- Dinamica interactiunii limfocite -tumora (model 3)

```
[t,y]=eul('funct2',[0 10],[2 5],.25)
plot(t,y(:,1), 'r')
hold on
plot(t,y(:,2),'g')
```

- Sol. aproximative in studiul dinamicii unei tumori (model 3)

```
clear all;
[teul,yeul]=eul('funct2',[0 10], [2 5],.25);
[trk2,yrk2]=rk2('funct2',[0 10], [2 5],.25);
[trk4,yrk4]=rk4('funct2',[0 10], [2 5],.25);
plot(teul,yeul(:,2),'g')
hold on
plot(trk2,yrk2(:,2),'c*')
plot(trk4,yrk4(:,2),'r')
```

S-au utilizat urmatoarele functii

```
funct.m
function w=funct(t,y)
D=20;
w=log(2)/D*y;
```

```
funct1.m
function w=funct1(t,y)
a=1.5;
a1=2;
w=(a1-a)*y;
```

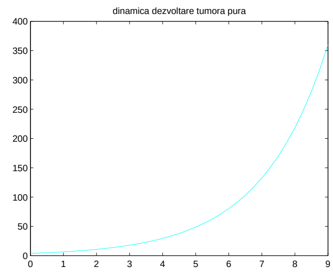

FIGURA 48

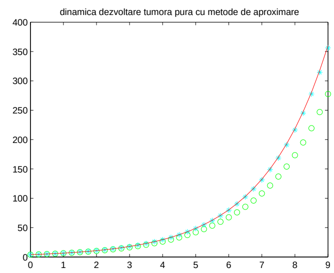

FIGURA 49

funct2.m
function w=funct2(t,y)
a=.2;
b=.05;
d=.29;
k=.1;
f=.001;
u=.1;
w(1)=a*y(1)-b*y(1)*y(2);
w(2)=d*y(1)*y(2)-k*y(1)-f*y(2)-u;

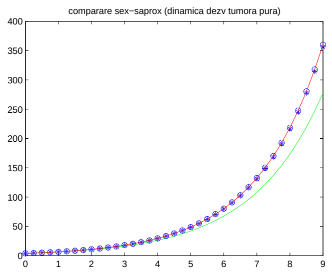

FIGURA 50

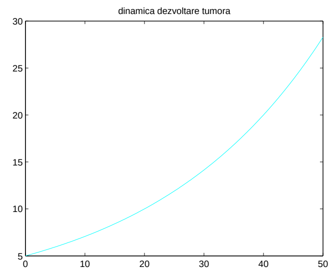

FIGURA 51

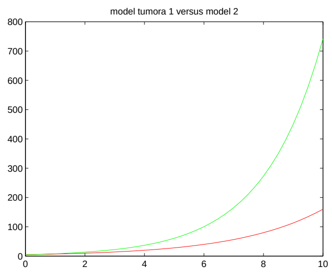

FIGURA 52

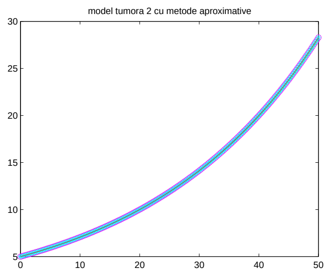

Figura 53

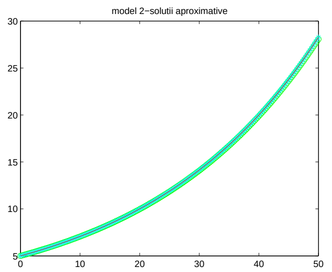

Figura 54

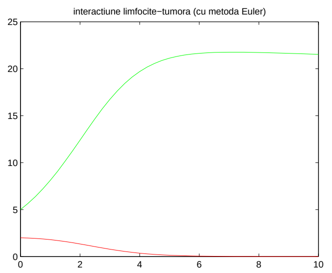

Figura 55

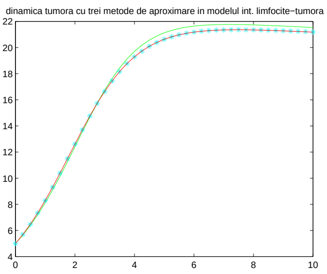
dinamica tumora cu trei metode de aproximare in modelul int. limfocite−tumora
22
20
18
16
14
12
10
8
6
4
0
2
4
6
8
10

FIGURA 56

Laborator 12

Tematica: modelare in epidemiologie

- Modelul SI

```
clear all;
[t,y]=eul('funct0',[0 150],[50 10],.5)
plot(t,y(:,1),'c')
hold on
plot(t,y(:,2),'g')
```

- Modelul SIR

```
[t,y]=eul('funct1' ,[0 10],[2 1 2], .5)
plot(t,y(:,1),'r')
hold on
plot(t,y(:,2),'g')
plot(t,y(:,3))
```

- Modelul SIR cu vaccinare

```
[t,y]=eul('funct2' ,[0 10],[2 1 2], .5)
plot(t,y(:,1),'r')
hold on
plot(t,y(:,2),'c')
plot(t,y(:,3),'g')
```

- Modelul SEIS

```
clear all;
S0=380;
E0=990;
I0=200;
N=S0+E0+I0;
[t,y]=rk4('funct' ,[0 10] ,[S0/N E0/N I0/N], .2)
plot(t,y(:,1),'r')
```

```
hold on
plot(t,y(:,2))
plot(t,y(:,3),'g')
```

- Modelul SEIR

```
clear all s0=500;
e0=700;
i0=100;
r0=50;
[t, y]=rk4('fseir',[0 60],[s0 e0 i0 r0],.1);
plot(t,y(:,1),'r')
hold on
plot(t,y(:,2),'g')
plot(t,y(:,3))
plot(t,y(:,4),'c')
```

- Dinamica infectati in modelul SEIR (diferite date initiale)

```
clear all s0=500;
e0=700;
i0=100;
r0=50;
[t, y1]=rk4('fseir',[0 260],[500 700 100 0],.1); [t, y2]=rk4('fseir',[0 260],[1000 800 500 0],.1); [t, y3]=rk4('fseir',[0 260],[2000 1000 900 0],.1); [t, y4]=rk4('fSEIR',[0 260],[3000 1200 1000 0],.1);
plot(t,y1(:,3),'r') hold on plot(t,y2(:,3),'g') plot(t,y3(:,3),'m') plot(t,y4(:,3),'c')
```

S-au utilizat urmatoarele functii:

```
funct.m
function w=funct(t,y)
r=10;
p=.4;
i=5;
l=3;
b=r*p;
d=1/l;
c=1/i;
```

```
w(1)=-b*y(1)*y(3)+c*y(3);
w(2)=b*y(1)*y(3)-d*y(2);
w(3)=d*y(2)-c*y(3);
```

funct0.m

```
function w=funct0(t,y)
r=.027;
b=.0099;
w(1)=-r*b*y(2)*y(1);
w(2)=r*b*y(2)*y(1);
```

funct1.m

```
function w=funct1(t,y)
a=.01;
b=.25;
c=1.2;
w(1)=-a*y(1)*y(2);
w(2)=a*y(1)*y(2)-b*y(2);
w(3)=b*y(2);
```

funct2.m

```
function w=funct2(t,y)
a=.01;
b=.25;
c=1.2;
w(1)=-a*y(1)*y(2)-c;
w(2)=a*y(1)*y(2)-b*y(2);
w(3)=b*y(2)+c;
```

fseir.m

```
function w=fseir(t,y)
b=.15;
d=0.0721;
c=0.0567;
w(1)=-b*y(1)*y(3)/(y(1)+y(2)+y(3)+y(4));
w(2)=b*y(1)*y(3)/(y(1)+y(2)+y(3)+y(4))-d*y(2);
w(3)=d*y(2)-c*y(3);
```

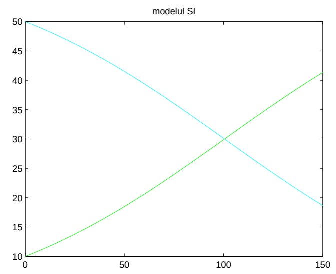

FIGURA 57

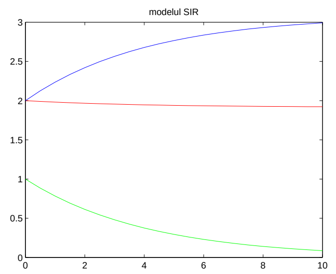

FIGURA 58

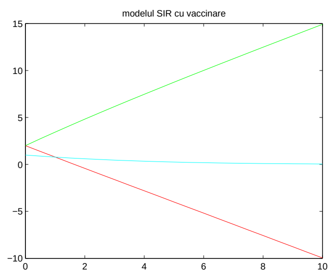

FIGURA 59

w(4)=c*y(3);

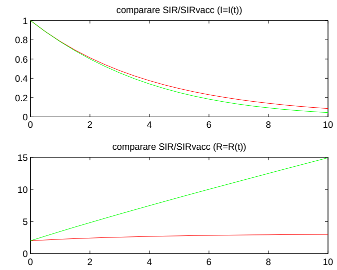

FIGURA 60

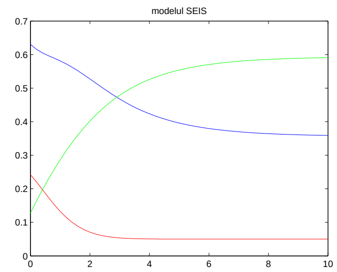

FIGURA 61

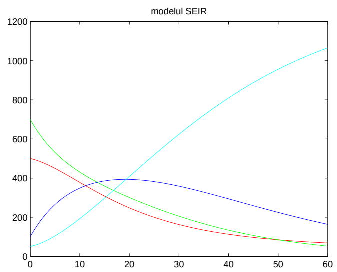

FIGURA 62

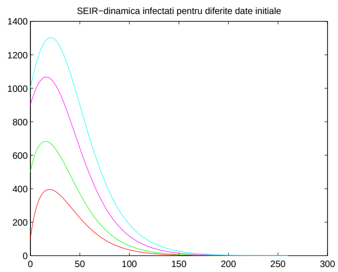
SEIR−dinamica infectati pentru diferite date initiale
1400
1200
1000
800
600
400
200
0
0
50
100
150
200
250
300

FIGURA 63

Laborator 13

Tematica: modelarea unor procese financiare

- Dinamica fond investitii I-solutie exacta

```
t=0:.5:10;
S0=1000;
a=.02;
S=S0.*exp(a*t);
plot(t,S,'c')
```

- Dinamica fond investitii II-sol.aprox

```
[trk4,yrk4]=rk4('funct',[0 10], 1000, .5)
plot(trk4,yrk4,'c')
```

- Dinamica fond investitii II-comparare solutii aproximative

```
[t,y1]=rk4('funct',[0 10], 1000, .5)
[t,y2]=rk2('funct',[0 10], 1000, .5)
[t,y3]=eul('funct',[0 10], 1000, .5)
plot(t,y1,'o')
hold on
plot(t,y2,'g')
plot(t,y3,'r')
```

- Dinamica investitie fond 1 versus fond 2

```
t=0:.2:10;
S0=1000;
a=.02;
S=S0.*exp(a*t);
[t,y]=rk4('funct',[0 10], 1000, .2)
plot(t,S,'g')
hold on
plot(t,y,'r')
```

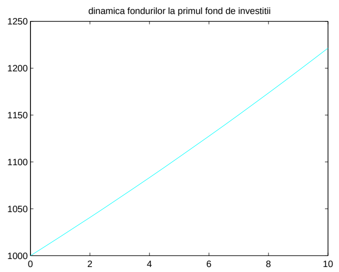

FIGURA 64

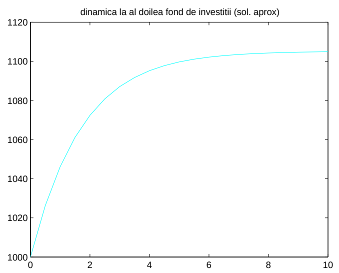

FIGURA 65

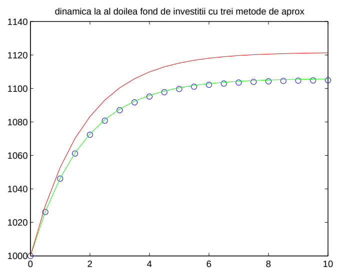

FIGURA 66

S-a utilizat urmatoarea functie:

funct.m
function w=funct(t,y) a0=.06; b=.6; w=a0*exp(-b*t)*y;

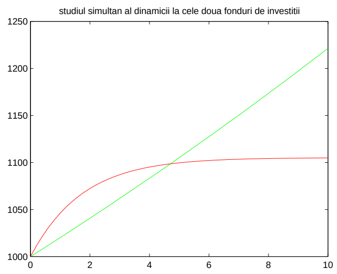
studiul simultan al dinamicii la cele doua fonduri de investitii
1250
1200
1150
1100
1050
1000
0
2
4
6
8
10

FIGURA 67

Laborator 14

Tematica: asupra reprezentarii solutiilor unor probleme antiplane
Fie $\Omega = (0,1) \times (0,1) \subset \mathbb{R}^2$.

Problema 1 (Probleme Dirichlet omogene).

$$\begin{cases} -\triangle u = f, & (x,y) \in \Omega, \\ u = 0, & (x,y) \in \partial\Omega. \end{cases} \tag{1}$$

Problema 2 (Probleme antiplane in deplasari tractiuni).

$$\begin{cases} -\triangle u = f, & (x,y) \in \Omega, \\ u = 0, & (x,y) \in \Gamma_0, \\ \frac{\partial u}{\partial \nu} = g, & (x,y) \in \Gamma_1. \end{cases} \tag{2}$$

Exemplul 1. Fie

$$f(x,y) = 25\pi^2 \sin(4\pi x) \cdot \sin(3\pi y).$$

Solutia Problemei 1 este

$$u(x,y) = \sin(4\pi x) \cdot \sin(3\pi y).$$

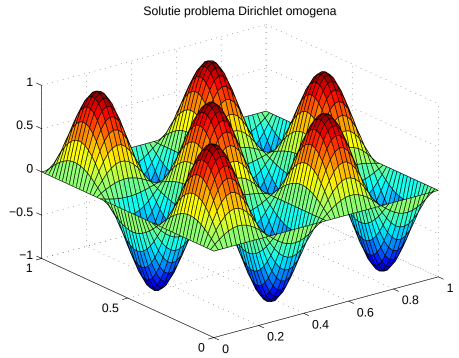

Figura 68

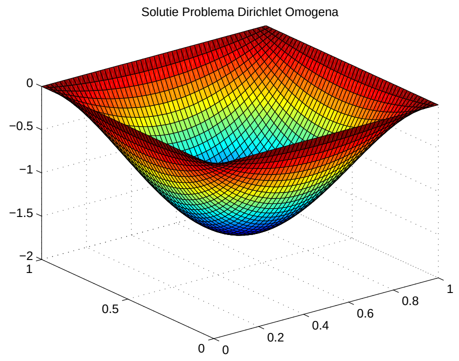

FIGURA 69

Exemplul 2. Fie

$$f(x,y) = 50 \cdot [x \cdot (1-x) + y \cdot (1-y)].$$

Solutia Problemei 1 este

$$u(x,y) = -25 \cdot x \cdot (1-x) \cdot y \cdot (1-y).$$

Exemplul 3. Fie

$$f(x,y) = \sin(\frac{\pi x}{2}) \cdot [-\frac{\pi^2}{4} \cdot y \cdot (1-y)^2 + 6 \cdot y - 4]$$

,

$$g(x,y) = 0$$

,

$$\Gamma_0 = \{(x,y) \in \Omega : x = 0, y = 0\}$$

,

$$\Gamma_1 = \{(x,y) \in \Omega : x = 1, y = 1\}.$$

Solutia Problemei 2 este

$$u(x,y) = \sin(\frac{\pi x}{2}) \cdot y \cdot (1-y)^2.$$

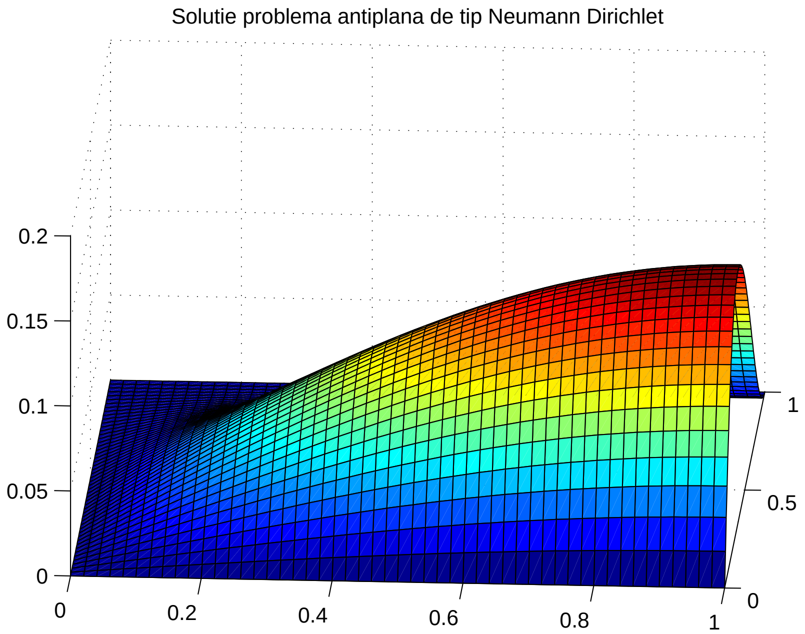

FIGURA 70

Exemplul 4. Fie

$$f(x,y) = \sin(\frac{\pi x}{2}) \cdot [\frac{\pi^2}{4} \cdot y \cdot (y-1)^2 - 6 \cdot y + 4]$$

,

$$g(x,y) = 0$$

,

$$\Gamma_0 = \{(x,y) \in \Omega : x = 0, y = 0\}$$

,

$$\Gamma_1 = \{(x,y) \in \Omega : x = 1, y = 1\}.$$

Solutia Problemei 2 este

$$u(x,y) = -\sin(\frac{\pi x}{2}) \cdot y \cdot (y-1)^2.$$

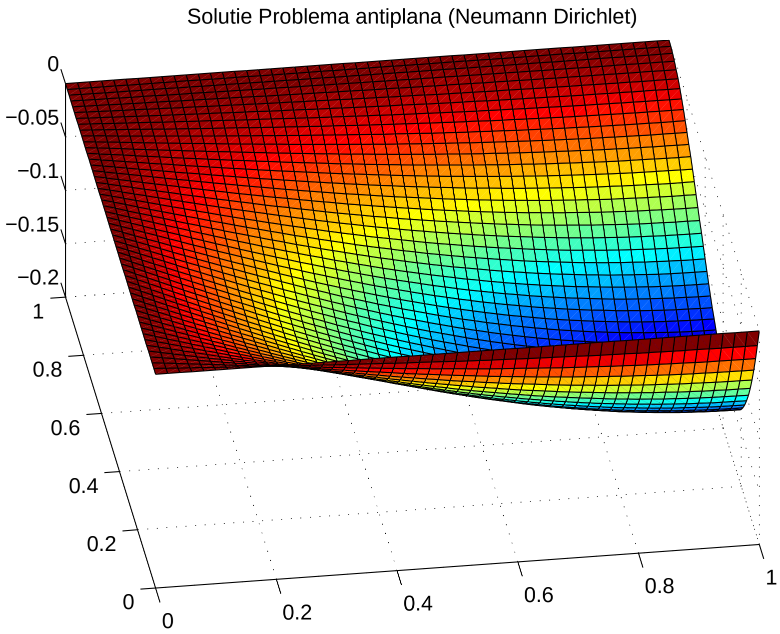
Solutie Problema antiplana (Neumann Dirichlet)
0
−0.05
−0.1
−0.15
−0.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

FIGURA 71

Bibliografie

1. Linda J.S. Allen, An introduction to mathematical biology, Pearson, Prentice Hall, 2007.
2. F. Brauer, C. Castillo-Chavez, Mathematical models in population biology and epidemiology, Springer 2001.
3. J. Beju, E.Sooś, P.P. Teodorescu, Tehnici de calcul vectorial cu aplicaţii. Tehnici de calcul tensorial euclidian cu aplicaţii, Ed tehnică, Bucuresti, 1977.
4. P.G. Ciarlet, *Mathematical Elasticity, Volume I: Three Dimensional Elasticity*, Studies in Mathematics and its Applications, Vol. 20, North-Holland, Amsterdam, 1988.
5. I. Doghri, *Mechanics of Deformable Solids*, Springer, Berlin, 2000.
6. G. Duvaut and J.-L. Lions, *Inequalities in Mechanics and Physics*, Springer-Verlag, Berlin, 1976.
7. M. Fan, M.Y. Li, K. Wang, Global stability of an SEIS epidemic model with recruitment and a varying total population size, Mathematical Biosciences 170(2001) 199–208, Elsevier.
8. Raluca-Mihaela Georgescu, Bifurcaie n dinamica biologic cu metode de teoria grupurilor, Seria Matematica Aplicata si Industriala, 2009.
9. P. Germain and P. Muller, *Introduction à la mécanique des milieux continus*, Masson, Paris, 1980.
10. M.E. Gurtin, *An Introduction to Continuum Mechanics*, Academic Press, New York, 1981.
11. W. Han and M. Sofonea, *Quasistatic Contact Problems in Viscoelasticity and Viscoplasticity*, Studies in Advanced Mathematics **30**, American Mathematical Society–International Press, 2002.
12. K.L. Johnson, *Contact Mechanics*, Cambridge University Press, Cambridge, 1987.
13. N. Kikuchi and J.T. Oden, *Contact Problems in Elasticity: A Study of Variational Inequalities and Finite Element Methods*, SIAM, Philadelphia, 1988.
14. A. Klarbring, A. Mikelic and M. Shillor, Frictional contact problems with normal compliance, *Int. J. Engng. Sci.* **26** (1988), 811–832.
15. A. Klarbring, A. Mikelič and M. Shillor, On friction problems with normal compliance, *Nonlinear Analysis* **13** (1989), 935–955.
16. T.A. Laursen, *Computational Contact and Impact Mechanics*, Springer, Berlin, 2002.
17. D. Lazăr, Calcul tensorial în Matematici Clasice şi Moderne, Ed tehnică, 1979, Bucuresti, Vol II.
18. J. Legrand, R.F. Grais, P.Y. Bolle, Understanding the dynamics of Ebola epidemics(2007)
19. L.E. Malvern, *Introduction to the Mechanics of a Continuum Medium*, Princeton-Hall, Inc, New Jersey, 1969.
20. A. Matei. Modelare matematica in elasticitate. Auxiliar pentru studenti (e-notes)
21. J.D. Murray, Mathematical biology: An introduction. Springer, 2002
22. J. Nečas and I. Hlaváček, *Mathematical Theory of Elastic and Elastico-Plastic Bodies: An Introduction*, Elsevier Scientific Publishing Company, Amsterdam, Oxford, New York, 1981.
23. P.D. Panagiotopoulos, *Inequality Problems in Mechanics and Applications*, Birkhäuser, Boston, 1985.
24. A. T. Porter, A path-specific approach to SEIR modeling (2012)
25. A. Prodeanu, Modele matematice în epidemiologie. Simulări numerice în MATLAB, Lucrare de Disertatie, UCV 2019, (coordonator Prof.univ.dr. A. Matei).
26. Alfio Quarteroni, Fausto Saleri, Paola Gervasio, Scientific Computing with Matlab and Octave, Springer 2014.

27. A. Quarteroni, F. Saleri, P. Gervasio, Calcul Scientifique, Cours, exercices corrigs et illustrations en Matlab et Octave, Springer, 2010.
28. A. Signorini, Sopra alcune questioni di elastostatica, *Atti della Società Italiana per il Progresso delle Scienze*, 1933.
29. D. Smith, L. Moore, The SIR Model for Spread of Disease- The Differentions (2004)
30. M. Sofonea, Problèmes non-lineaires dans la théorie de l'élasticité, Cours de Magister de Mathématiques Appliquées a l'Université de Sétif, Algérie, 1993.
31. M. Sofonea, Problèmes Mathématiques en Elasticité et Viscoplasticité,non-lineaires dans la théorie de l'élasticité, Cours de DEA de Mathématiques Appliquées a l'Université Blaise Pascal(Clermont-Ferrand II), Laboratoire de Mathématiques Appliquées et URA-CNRS 1501, 1993.
32. M. Shillor, M. Sofonea and J.J. Telega, *Models and Analysis of Quasistatic Contact*, Lecture Notes in Physics **655**, Springer, Berlin, 2004.
33. M. Sofonea, W. Han and M. Shillor, *Analysis and Approximation of Contact Problems with Adhesion or Damage*, Pure and Applied Mathematics **276**, Chapman-Hall/CRC Press, New York, 2006.
34. M. Sofonea, Problèmes mathématiques en elasticité et en viscoplasticité, CNRS 1501, Laboratoire de Mathématiquesappliquées, Université Blaise Pascal Clermont Ferrand, 1993.
35. M. Sofonea, A. Matei, *Variational Inequalities with Applications. A study of Antiplane Frictional Contact Problems*, Springer, 2009.
36. M. Sofonea, A. Matei, Mathematical Models in Contact Mechanics, Cambridge University Press, 2012.
37. R. Temam and A. Miranville, *Mathematical Modeling in Continuum Mechanics*, Cambridge University Press, Cambridge, 2001.
38. M. Trifan, Dinamica si bifurcatie in studiul matematic al cancerului, Seria Matematica Aplicata si Industriala, Editura Pamantul 2006.
39. P. Wriggers, *Computational Contact Mechanics*, Wiley, Chichester, 2002.
40. "21-124 Modeling with differential equations - CMU Math"
`www.math.cmu.edu/~handron/21_124/`
41. "Differential Equations in Economics"
`u.math.biu.ac.il/~shnider/Course_Notes/ODE-economics-chap1.pdf`
42. Modeling with ODE - TAMU Math
`www.math.tamu.edu/~phoward/m442/modode.pdf`
43. `https://en.wikipedia.org/wiki/Epidemic_model`
44. `https://en_wikipedia.org/wiki/Compartmental_mode_in_epidemiology`
45. demonstrations.wolfram.com/SIREpidemiologic.Dynamic
46. mysite.science.uottawa.ca/rsmith43/mat4996/Epidemic.pdf