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Explosive solutions of semilinear elliptic systems with gradient term

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Abstract. We study the existence of boundary blow-up solutions to the nonlinear elliptic system $\Delta u + |\nabla u| = p(|x|)f(v)$, $\Delta v + |\nabla v| = q(|x|)g(u)$ in Ω. Here Ω is either a bounded domain in $\mathbb{R}^N$ or it denotes the whole space. The nonlinearities f and g are positive and continuous, while the nonnegative potentials p and q are continuous and satisfy appropriate growth conditions at infinity. We show that boundary blow-up positive solutions fail to exist if f and g are sublinear. This result holds both if Ω is bounded, and if Ω is the whole space but p and q have slow decay at infinity. We establish the existence of infinitely many entire blow-up solutions in the case where p and q are of fast decay and if f and g satisfy a sublinear type growth condition at infinity.

Soluciones explosivas de sistemas elípticos semilineales con términos gradientes

Resumen. Estudiamos la existencia de soluciones del sistema elíptico no lineal $\Delta u + |\nabla u| = p(|x|)f(v)$, $\Delta v + |\nabla v| = q(|x|)g(u)$ en Ω que explotan en el borde. Aquí Ω es un dominio acotado de $\mathbb{R}^N$ o el espacio total. Las nolinealidades f y g son funciones continuas positivas mientras que los potenciales p y q son funciones continuas que satisfacen apropiadas condiciones de crecimiento en el infinito. Demostramos que las soluciones explosivas en el borde dejan de existir si f y g son sublineales. Esto se tiene o bien si Ω es acotado o cuando Ω es el espacio total pero p y q decaen lentamente en el infinito. Mostramos la existencia de infinitas soluciones enteras explosivas cuando p y q decaen rápidamente y cuando f y g satisfacen una condición de tipo sublineal en el infinito.

1. Introduction and the main results

Existence and nonexistence of solutions of the semilinear elliptic system

$$\begin{cases} \Delta u = f(x, u, v) & \text{in } \Omega, \\ \Delta v = g(x, u, v) & \text{in } \Omega \end{cases} \tag{1}$$

have received much attention recently. See, for example, Chen and Lu [2], Cîrstea and Rădulescu [4], Clément, Manásevich and Mitidieri [5], Dalmasso [6], De Figueiredo and Jianfu [7], Lair and Shaker [14], Serrin and Zou [18, 19], Yarur [20], Wang and Wood [21], and the references therein. Most of these results have to do with the nonexistence of positive solutions, the existence of radial solutions, or the asymptotic behavior of solutions.

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We are concerned in this paper with the study of positive solutions to the following class of semilinear elliptic systems with gradient term

$$\begin{cases} \Delta u + |\nabla u| = p(|x|)f(v) & \text{in } \Omega, \\ \Delta v + |\nabla v| = q(|x|)g(u) & \text{in } \Omega, \end{cases} \tag{2}$$

where $\Omega \subset \mathbb{R}^N$ ($N \geq 1$) denotes either a bounded open set in $\mathbb{R}^N$ or the whole of $\mathbb{R}^N$. Throughout this paper we assume that $p, q \not\equiv 0$ are nonnegative Hölder functions. We also assume that f and g are Hölder, positive and non-decreasing functions on $(0, \infty)$.

We are mainly interested in finding properties of *large (explosive, blow-up) solutions* of (2), that is positive solutions (u, v) satisfying $u(x) \to +\infty$ and $v(x) \to +\infty$ as $\operatorname{dist}(x, \partial\Omega) \to 0$ (if Ω is bounded), or $u(x) \to +\infty$ and $v(x) \to +\infty$ as $|x| \to \infty$ (if $\Omega = \mathbb{R}^N$). In the latter case such solutions are called *entire large (explosive, blow-up) solutions*. A geometric motivation in that sense can be found in [3, 12, 15]. We also point out the pioneering work of Keller [10] and Osserman [16].

The corresponding equation that leads us to the system (2) is

$$\Delta u + |\nabla u|^a = p(x)f(u), \quad x \in \Omega,\, 0 < a \leq 2,$$

which was treated in [1, 8] (in the case where Ω is bounded) and in [9, 13] (for $\Omega = \mathbb{R}^N$). Problems of this type arise in stochastic control theory and have been first studied in Lasry and Lions [11]. The corresponding parabolic equation was considered in Quittner [17]. In terms of the dynamic programming approach, an explosive solution of (2) corresponds to a value function (or Bellman function) associated to an infinite exit cost (see [11]).

Our first result asserts that if Ω is bounded and if both f and g are sublinear at infinity, then problem (2) has no positive boundary blow-up solution. More precisely, the following hold

Theorem 1 *Suppose $\Omega \subset \mathbb{R}^N$ is a bounded domain and f, g satisfy*

$$\max\left\{\sup_{t\geq 1}\frac{f(t)}{t},\ \sup_{t\geq 1}\frac{g(t)}{t}\right\} < +\infty. \tag{A_1}$$

Then problem (2) has no positive large solution.

The same conclusion holds if $\Omega = \mathbb{R}^N$, but under natural additional assumptions related to the behavior of p and q at infinity. In order to state the result in this case, let us first define, for any $r \geq 0$,

$$P(r) = \frac{\displaystyle\int_0^r e^t t^{N-1} p(t)dt}{e^r r^{N-1}}, \quad Q(r) = \frac{\displaystyle\int_0^r e^t t^{N-1} q(t)dt}{e^r r^{N-1}}. \tag{3}$$

Theorem 2 *Let $\Omega = \mathbb{R}^N$. Assume that (A_1) holds and*

$$\int_1^\infty P(r)dr < +\infty, \quad \int_1^\infty Q(r)dr < +\infty. \tag{4}$$

Then problem (2) has no positive entire large solution.

Theorem 3 *Let $\Omega = \mathbb{R}^N$. Assume that*

$$\int_1^\infty P(r)dr = +\infty, \quad \int_1^\infty Q(r)dr = +\infty. \tag{5}$$

If

$$\lim_{t\to\infty}\frac{f(ag(t))}{t} = 0, \quad \textit{for all constants } a \geq 1, \tag{A_2}$$

then problem (2) has infinitely many positive entire large solutions.

We point out that Condition (A_2) has been introduced in [4].

Remark 1 Using the fact that

$$\int_0^r e^r t^k dt = k! e^r \sum_{s=1}^{k} (-1)^{k-s} \frac{t^s}{s!} \quad \text{for all integers } \ k \geq 1, \tag{6}$$

we observe that the following functions verify (4) or (5):
(i) condition (4) holds provided that $p(t) = \dfrac{1}{1+t^\gamma}$, $\gamma > 1$ and $q(t) = \dfrac{1}{(1+t^2)^\theta}$, $\theta > \dfrac{1}{2}$.
(ii) condition (5) holds provided that $p(t) = t^\gamma,\ q(t) = t^\theta,\ \gamma, \theta \geq 0.$ ■

Remark 2 We give in what follows some examples of nonlinearities f and g that satisfy (A_2):
(i) $f(t) = \sum_{j=1}^{l} a_j t^{\gamma_j},\ g(t) = \sum_{k=1}^{m} b_k t^{\theta_k},\ t \geq 0$ with $a_j, b_k, \gamma_j, \theta_k > 0$ and $\gamma\theta < 1$, where $\gamma = \max_{1\leq j\leq l} \gamma_j$, $\theta = \max_{1\leq k\leq m} \theta_k$.
(ii) $f(t) = (1+t^{\gamma_1})^{\gamma_2}$, $g(t) = (1+t^{\theta_1})^{\theta_2}$, where $\gamma_1, \gamma_2, \theta_1, \theta_2 > 0$ and $\gamma_1\gamma_2\theta_1\theta_2 < 1$.
(iii) $f(t) = \ln(1+t^\gamma)$, $g(t) = \ln(1+t^\theta)$, $\ \gamma, \theta > 0$.
(iv) $f(t) = \ln(1+t^\gamma)$, $g(t) = e^{t^\theta}$, $\gamma > 0$, $\theta \in (0,1)$. ■

2. Proof of Theorem 1

Suppose that (u, v) is a positive large solution of (2) and let $w(x) = \ln(1 + u(x) + v(x)),\ x \in \Omega$. Then w is a positive function and $w(x) \to \infty$ as $\operatorname{dist}(x, \partial\Omega) \to 0$. A simple calculation yields

$$\Delta w = \frac{\Delta u + \Delta v}{1+u+v} - \frac{\sum_{i=1}^{N} (u_{x_i} + v_{x_i})^2}{(1+u+v)^2} \quad \text{in } \Omega.$$

Taking into account the assumption (A_1) we have

$$\begin{aligned}
\Delta w \quad & \leq \frac{\Delta u + \Delta v}{1+u+v} \\
& \leq \frac{\|p\|_{L^\infty(\Omega)} f(v) + \|q\|_{L^\infty(\Omega)} g(u)}{1+u+v} \\
& \leq \left(\|p\|_{L^\infty(\Omega)} + \|q\|_{L^\infty(\Omega)}\right) \frac{f(v)+g(u)}{1+u+v} \\
& \leq \left(\|p\|_{L^\infty(\Omega)} + \|q\|_{L^\infty(\Omega)}\right) \left(\frac{f(1+v)}{1+v} + \frac{g(1+u)}{1+u}\right) \leq K,
\end{aligned}$$

for some constant $K > 0$. Hence

$$\Delta(w(x) - K|x|^2) < 0, \quad \text{for all } \ x \in \Omega.$$

Let $z(x) = w(x) - K|x|^2,\ x \in \Omega$. Then

$$\Delta z < 0 \quad \text{in } \Omega \tag{7}$$

and

$$z(x) \to \infty \quad \text{as } \operatorname{dist}(x, \partial\Omega) \to 0. \tag{8}$$

Fix $x_0 \in \Omega$ and $M > 0$. At this point, to reach a contradiction we will show that $z(x_0) > M$. Suppose $z(x_0) \leq M$. For all $\delta > 0$, we set

$$\Omega_\delta = \{x \in \Omega \mid \operatorname{dist}(x, \partial\Omega) > \delta\}.$$

Since $z(x) \to \infty$ as $\mathrm{dist}(x, \partial\Omega) \to 0$, we can choose $\delta > 0$ such that $z(x) > M$ for all $x \in \Omega \setminus \Omega_\delta$. Obviously, $x_0 \in \Omega_\delta$. Moreover, $M - z(x_0) \geq 0$ and $(M - z)\,|_{\partial\Omega_\delta} \leq 0$. Therefore we can find $\bar{x} \in \Omega_\delta$ such that

$$\max_{\overline{\Omega}_\delta}(M - z(x)) = M - z(\bar{x}) \leq 0.$$

It follows that $\Delta(M - z)(\bar{x}) \leq 0$, that is $\Delta z(\bar{x}) \geq 0$ which contradicts (7). Hence (2) has no positive large solutions. This completes the proof. ■

Remark 3 We can employ the same method as above to show that the system

$$\begin{cases} \Delta u + |\nabla v| = p(|x|)f(v) & \text{in } \Omega, \\ \Delta v + |\nabla u| = q(|x|)g(u) & \text{in } \Omega, \end{cases}$$

has no positive large solutions if f and g satisfy (A_1). ■

3. Proof of Theorem 2

Arguing by contradiction, let us assume that the system (2) has the positive entire large solution (u, v). Consider the spherical average of u and v defined by

$$\bar{u}(r) = \frac{1}{c_N r^{N-1}} \int\limits_{|x|=r} u(x) d\sigma_x, \quad r \geq 0 \tag{9}$$

$$\bar{v}(r) = \frac{1}{c_N r^{N-1}} \int\limits_{|x|=r} v(x) d\sigma_x, \quad r \geq 0 \tag{10}$$

where c_N is the surface area of the unit sphere in $\mathbb{R}^N$. Since u and v are positive entire large solutions it follows that $\bar{u}, \bar{v}$ are positive and $\lim\limits_{r\to\infty} \bar{u}(r) = \lim\limits_{r\to\infty} \bar{v}(r) = +\infty$. By the change of variable $x \to ry$, we have

$$\bar{u}(r) = \frac{1}{c_N} \int\limits_{|y|=1} u(ry)\, d\sigma_y, \quad r \geq 0$$

and

$$\bar{u}'(r) = \frac{1}{c_N} \int\limits_{|y|=1} \nabla u(ry) \cdot y\, d\sigma_y, \quad r \geq 0. \tag{11}$$

The above relation may be rewritten as

$$\bar{u}'(r) = \frac{1}{c_N} \int\limits_{|y|=1} \frac{\partial u}{\partial r}(ry)\, d\sigma_y = \frac{1}{c_N r^{N-1}} \int\limits_{|x|=r} \frac{\partial u}{\partial r}(x)\, d\sigma_x,$$

that is

$$\bar{u}'(r) = \frac{1}{c_N r^{N-1}} \int\limits_{|x|=r} \Delta u(x)\, d\sigma_x, \quad \text{for all } r \geq 0. \tag{12}$$

Similarly we have

$$\bar{v}'(r) = \frac{1}{c_N r^{N-1}} \int\limits_{|x|=r} \Delta v(x)\, d\sigma_x, \quad \text{for all } r \geq 0. \tag{13}$$

Due to the presence of the gradient term in (2), we cannot infer that $\Delta u \geq 0$ in $\mathbb{R}^N$ and so we do not know if $\bar{u}' \geq 0$ (or $\bar{v}' \geq 0$) in $[0, \infty)$. In order to overcome this lack of monotonicity, set

$$U(r) = \max_{0 \leq t \leq r} \bar{u}(r), \quad V(r) = \max_{0 \leq t \leq r} \bar{v}(r). \tag{14}$$

Now it is easy to see that U, V are positive and non-decreasing functions. Moreover $U \geq \bar{u}$, $V \geq \bar{v}$ and $U(r), V(r) \to +\infty$ as $r \to \infty$.

By (A_1), that there exists $M > 0$ such that

$$\max\{f(t), g(t)\} \leq M(1+t), \quad \text{for all } t \geq 0. \tag{15}$$

Now (11), (12) and (15) lead to

$$\begin{aligned}
\bar{u}'' + \frac{N-1}{r}\bar{u}' + \bar{u}' &\leq \frac{1}{c_N r^{N-1}} \int_{|x|=r} [\Delta u(x) + |\nabla u|(x)]\, d\sigma_x \\
&= p(r) \frac{1}{c_N r^{N-1}} \int_{|x|=r} f(v(x)) d\sigma_x \\
&\leq Mp(r) \frac{1}{c_N r^{N-1}} \int_{|x|=r} (1 + v(x))\, d\sigma_x \\
&= Mp(r)\,(1 + \bar{v}(r)) \\
&\leq Mp(r)\,(1 + V(r)),
\end{aligned}$$

for all $r \geq 0$. It follows that

$$\left(r^{N-1} e^r \bar{u}'\right)' \leq Me^r r^{N-1} p(r)\,(1 + V(r)) \quad \text{for all } r \geq 0.$$

So, for all $r \geq r_0 > 0$,

$$\begin{aligned}
\bar{u}(r) &\leq \bar{u}(r_0) + M \int_{r_0}^{r} e^{-t} t^{1-N} \int_0^t e^s s^{N-1} p(s)(1 + V(s)) ds dt \\
&\leq \bar{u}(r_0) + M \int_{r_0}^{r} e^{-t} t^{1-N} (1 + V(t)) \int_0^t e^s s^{N-1} p(s) ds dt \\
&\leq \bar{u}(r_0) + M(1 + V(r)) \int_{r_0}^{r} e^{-t} t^{1-N} \int_0^t e^s s^{N-1} p(s) ds dt,
\end{aligned}$$

that is

$$\bar{u}(r) \leq \bar{u}(r_0) + M(1 + V(r)) \int_{r_0}^{r} P(t) dt, \quad \text{for all } r \geq r_0 \geq 0. \tag{16}$$

Since $\int_1^\infty P(r) dr < \infty$ and $\int_1^\infty Q(r) dr < \infty$, we can choose $r_0 \geq 1$ such that

$$\max\left\{\int_{r_0}^\infty P(r) dr, \int_{r_0}^\infty Q(r) dr\right\} < \frac{1}{2M}. \tag{17}$$

From (14) and the fact that $\lim_{r\to\infty} \bar{u}(r) = \lim_{r\to\infty} \bar{v}(r) = \infty$, we can find $r_1 \geq r_0$ such that

$$U(r) = \max_{r_0 \leq t \leq r} \bar{u}(r), \;\; V(r) = \max_{r_0 \leq t \leq r} \bar{v}(r), \quad \text{for all } r \geq r_1. \tag{18}$$

Thus (16) and (18) yield

$$U(r) \leq \bar{u}(r_0) + M(1 + V(r)) \int_{r_0}^{r} P(t) dt, \quad \text{for all } r \geq r_1.$$

Furthermore, by (17) we obtain

$$U(r) \leq \bar{u}(r_0) + \frac{1+V(r)}{2} \quad \text{for all } \ r \geq r_1,$$

and so

$$U(r) \leq C_1 + \frac{1}{2}V(r) \quad \text{for all } \ r \geq r_1, \tag{19}$$

where $C_1 = \dfrac{1}{2} + \bar{u}(r_0) > 0$. In a similar way we get

$$V(r) \leq C_2 + \frac{1}{2}U(r) \quad \text{for all } \ r \geq r_1, \tag{20}$$

By addition, (19) and (20) lead to

$$U(r) + V(r) \leq 2(C_1 + C_2) \quad \text{for all } \ r \geq r_1. \tag{21}$$

This means that U and V are bounded and so u and v are bounded which is a contradiction. It follows that (2) has no positive entire large solutions and the proof is now complete. ■

4. Proof of Theorem 3

We start by showing that (2) has positive radial solutions. On this purpose we fix $a > 0$ and $b > 0$ and we show that the system

$$\begin{cases} u'' + \dfrac{N-1}{r}u' + u' = p(r)f(v(r)), & r > 0, \\ v'' + \dfrac{N-1}{r}u' + v' = q(r)g(u(r)), & r > 0, \\ u', v' \geq 0 \quad \text{on } [0,\infty), \\ u(0) = a > 0, \ v(0) = b > 0, \end{cases} \tag{22}$$

has solutions. Then $U(x) = u(|x|)$, $V(x) = v(|x|)$ are positive solutions of (2).
Integrating (22) we have

$$u(r) = a + \int_0^r e^{-t}t^{1-N}\int_0^t e^s s^{N-1}p(s)f(v(s))dsdt \quad \forall\, r \geq 0, \tag{23}$$

$$v(r) = b + \int_0^r e^{-t}t^{1-N}\int_0^t e^s s^{N-1}q(s)g(u(s))dsdt \quad \forall\, r \geq 0. \tag{24}$$

Define $v_0 \equiv b$ and let $(u_k)_{k\geq 1}, (v_k)_{k\geq 1}$ given by

$$u_k(r) = a + \int_0^r e^{-t}t^{1-N}\int_0^t e^s s^{N-1}p(s)f(v_{k-1}(s))dsdt \quad \forall\, r \geq 0, \tag{25}$$

$$v_k(r) = b + \int_0^r e^{-t}t^{1-N}\int_0^t e^s s^{N-1}q(s)g(u_k(s))dsdt \quad \forall\, r \geq 0. \tag{26}$$

Since $v_1(r) \geq b$, it follows that $u_2(r) \geq u_1(r)$ for all $r \geq 0$ which yields $v_2(r) \geq v_1(r)$ and so $u_3(r) \geq u_2(r)$ for all $r \geq 0$. Repeating such arguments we deduce that

$$u_k(r) \leq u_{k+1}(r) \ \text{ and } \ v_k(r) \leq v_{k+1}(r), \quad \text{for all } \ r > 0, k \geq 1.$$

Let us now prove that the non-decreasing sequences $(u_k)_{k\geq 1}$ and $(v_k)_{k\geq 1}$ are bounded from above on bounded sets. We first observe that (25) and (26) yield

$$u_k(r) \leq u_{k+1}(r) \leq a + f(v_k(r)) \int_0^r P(t)dt, \quad \forall\, r \geq 0,\, k \geq 1 \tag{27}$$

and

$$v_k(r) \leq b + g(u_k(r)) \int_0^r Q(t)dt, \quad \forall\, r \geq 0,\, k \geq 1 \tag{28}$$

Let $R > 0$ be arbitrary. From (27) and (28) we get

$$u_k(R) \leq a + f\left(b + g(u_k(R)) \int_0^R Q(t)dt\right) \int_0^R P(t)dt, \quad \forall\, k \geq 1.$$

This imply

$$1 \leq \frac{a}{u_k(R)} + \frac{f\left(b + g(u_k(R)) \int_0^R Q(t)dt\right)}{u_k(R)} \int_0^R P(t)dt, \quad \forall\, k \geq 1. \tag{29}$$

Taking into account the monotonicity of $(u_k(R))_{k\geq 1}$, there exists $L(R) := \lim_{k\to\infty} u_k(R)$.
We claim that $L(R)$ is finite. Indeed, if not, we let $k \to \infty$ in (29) and the assumption (A_2) leads us to a contradiction. Thus $L(R)$ is finite. Since u_k, v_k are increasing functions, it follows that the map $(0,\infty) \ni R \longmapsto L(R)$ is non-decreasing on $(0,\infty)$ and

$$u_k(r) \leq u_k(R) \leq L(R), \quad \forall\, r \in [0, R],\, \forall\, k \leq 1,$$

$$v_k(r) \leq b + g(L(R)) \int_0^R Q(t)dt, \quad \forall\, r \in [0, R],\, \forall\, k \leq 1.$$

Furthermore, there exists $\lim_{R\to\infty} L(R) = \bar{L} \in (0,\infty]$ and the sequences $(u_k)_{k\geq 1}$ and $(v_k)_{k\geq 1}$ are bounded from above on bounded sets.

Let $u(r) := \lim_{k\to\infty} u_k(r),\ v(r) := \lim_{k\to\infty} v_k(r)$ for all $r \geq 0$. By standard elliptic regularity theory we deduce that (u, v) is a positive solution of (22).

In order to conclude the proof, it is enough to show that (u, v) is a large solution of (22). Let us remark that (23), (24) imply

$$u(r) \geq a + f(b) \int_0^r P(t)dt, \quad \forall\, r \geq 0,$$

$$v(r) \geq b + g(a) \int_0^r Q(t)dt, \quad \forall\, r \geq 0.$$

Since f, g are positive functions and p, q satisfy (5) we can conclude that (u, v) is a large solution of (22) and so (U, V) is a positive entire large solution of (2). Hence any large solution of (22) provides a positive entire large solution (U, V) of (2) with $U(0) = a$ and $V(0) = b$. Since $(a, b) \in (0,\infty) \times (0,\infty)$ was chosen arbitrarily, it follows that (2) has infinitely many positive entire large solutions. The proof of theorem is now complete. ■

Remark 4 The condition (5) is sufficient but not necessary for the existence of positive entire large solutions for (2). Indeed, let us consider $f(t) = \sqrt{t},\ g(t) = t,\ p(r) = 4\,\dfrac{r^3 + (N+2)r^2}{\sqrt{r^2+1}},\ q(r) = 2\,\dfrac{r+N}{r^4+1}$.

Using (6) we get $\int_1^\infty P(r)dr = +\infty$ and $\int_1^\infty Q(r)dr < +\infty$. However, the corresponding system to (2) is

$$\begin{cases} \Delta u + |\nabla u| = 4\,\dfrac{|x|^3 + (N+2)|x|^2}{\sqrt{|x|^2+1}} \cdot \sqrt{v} & \text{in } \mathbb{R}^N, \\ \Delta v + |\nabla v| = 2\,\dfrac{|x| + N}{|x|^4+1} \cdot u & \text{in } \mathbb{R}^N, \end{cases}$$

which has the positive entire large solution $(|x|^4+1, |x|^2+1)$. ■

References

[1] Bandle, C. and Giarrusso, E. (1996). Boundary blow-up for semilinear elliptic equations with nonlinear gradient terms, *Adv. Differential Equations*, **1**, 133–150.

[2] Chen, S. and Lu, G. (1999). Existence and nonexistence of positive solutions for a class of semilinear elliptic systems, *Nonlinear Anal.*, **38**, 919–932.

[3] Cheng, K.-S. and Ni, W.-M. (1991). On the structure of the conformal gaussian curvature equation in $\mathbb{R}^2$, *Duke Math. J.,* **62**, 721-737.

[4] Cîrstea, F. and Rădulescu, V. (2002). Entire solutions blowing up at infinity for semilinear elliptic systems, *J. Math. Pures Appl.,* **81**, 827–846.

[5] Clément, P., Manásevich, R. and Mitidieri, E. (1993). Positive solutions for a quasilinear system via blow up, *Comm. Partial and Differential Equations,* **18**, 2071–2106.

[6] R. Dalmasso, Existence and uniqueness of positive solutions of semilinear elliptic systems, *Nonlinear Anal.,* **39** (2000), 559-568.

[7] De Figueiredo, D. G. and Jianfu, Y. (1998). Decay, symmetry and existence of solutions of semilinear elliptic systems, *Nonlinear Anal.,* **33**, 211–234.

[8] Giarrusso, E. (2000). On blow up solutions of a quasilinear elliptic equation, *Math. Nachr.,* **213**, 89–104.

[9] Ghergu, M., Niculescu, C. and Rădulescu, V. (2002). Existence solutions of elliptic equations with absorption and nonlinear gradient term, *Proc. Indian Acad. Sci. (Math. Sci.),* **112**, 441–451.

[10] Keller, J. B. (1957). On solution of $\Delta u = f(u)$, *Comm. Pure Appl. Math.,* **10**, 503–510.

[11] Lasry, J. M. and Lions, P.-L. (1989). Nonlinear elliptic equations with singular boundary conditions and stochastic control with state constraints; the model problem, *Math. Ann.,* **283**, 583–630.

[12] Loewner, C. and Nirenberg, L. (1974). Partial differential equations invariant under conformal or projective transformations, in *Contributions to Analysis* (L. Ahlfors et al., Eds.), Academic Press, New York, 245–272.

[13] Lair, A. V. and Shaker, A. W. (1996). Entire solution of a singular semilinear elliptic problem, *J. Math. Anal. Appl.,* **200**, 498–505.

[14] Lair, A. V. and Shaker, A. W. (2000). Existence of entire large positive solutions of semilinear elliptic systems, *J. Differential Equations,* **164**, 380–394.

[15] Nirenberg, L. (1976). Nonlinear differential equations invariant under certain geometric transformations, *Sympos. Math.* vol. 18, Academic Press, New York, 399–405.

[16] Osserman, R. (1957). On the inequality $\Delta u \geq f(u)$, *Pacific J. Math.,* **7**, 1641–1647.

[17] Quittner, P. (1991). Blow-up for semilinear parabolic equations with a gradient term, *Math. Meth. Appl. Sci.,* **14**, 413–417.

[18] Serrin, J. and Zou, H. (1996). Nonexistence of positive solutions of Lane-Emden systems, *Differential Integral Equations*, **9**, 635–653.

[19] Serrin, J. and H. Zou, H. (1998). Existence of positive entire solutions of elliptic Hamiltonian systems, *Comm. Partial Differential Equations*, **23**, 577–599.

[20] Yarur, C. (1998). Existence of continuous and singular ground states for semilinear elliptic systems, *Electron. J. Differential Equations*, **1**, 1–27.

[21] Wang, X. and Wood, A. W. (2002). Existence and nonexistence of entire positive solutions of semilinear elliptic systems, *J. Math. Anal. Appl.*, **267**, 361–368.

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