UNIVERSITY OF CRAIOVA Faculty of Mathematics and Natural Sciences Department of mathematics Fundamental domain : Mathematics Specialization: Mathematics Education form: full time Duration of studies : 3 years Academic year : 2013-2014

Subject : *Probability theory and mathematical statistics* Code: M3503 Instructor : Lect.dr. Monica Roșiu Number of credits : 5 Year/Semester : Year III, Semester I Number of hours/week : 2 Classes +2 Seminar classes Number of weeks : 14 Type discipline: Fundamental

Objectives: The purpose of this course is to introduce the basics of probability theory and mathematical statistics, in order to form a solid theoretical foundation for the students' future professions. This course will explain the origins of the theoretical formulas and the practical implications of the probabilistic and statistical formulas and build a series of techniques that will be useful in the study of mathematical models involving uncertainty.

Content :

1. Events and their probabilities (1 course)

- 1.1. Sample space and Events.
- 1.2. Definitions of probability.
- 1.3. Some simple propositions.
- 1.4. Probability as a continuous set function.

2. Conditional Probability and Independence (2 courses)

- 2.1. Conditional Probabilities
- 2.2. Multiplication rule. The total probability formula.
- 2.3. Bayes's Formula.
- 2.4. Independent Events
- 2.5. $P(\cdot | B)$ is a probability.

3. Discrete Random Variables (2 courses)

- 3.1. Random Variables. The cumulative distribution function. Properties.
- 3.2. Discrete Random Variables. The probability mass function. The cumulative distribution function
- 3.3. Expected Value.
- 3.4. Expectation of a Function of a Random Variable.
- 3.5. Variance.

3.6. The Bernoulli and Binomial Random Variables. Properties. Computing the Binomial Distribution Function, the expected value and the variance.

3.7. The Poisson Random Variable. Computing the Poisson Distribution

Function, the expected value and the variance.

3.8. The Geometric Random Variable.

- 3.9. The Negative Binomial Random Variable.
- 3.10. The Hypergeometric Random Variable.
- 3.11. Expected Value of Sums of Random Variables.

4. Continuous Random Variables (1 course)

- 4.1. The probability density function. The cumulative distribution function.
- 4.2. Expectation and Variance of Continuous Random Variables.
- 4.3. The Uniform Random Variable.

4.4. Normal Random Variables. The Normal Approximation to the Binomial Distribution.

4.5. Exponential Random Variables.

4.6. Other Continuous Distributions. TheGamma Distribution. TheWeibull Distribution. TheCauchy Distribution. The Beta Distribution.

4.7. The Distribution of a Function of a Random Variable.

5. Jointly Distributed Random Variables (2 courses)

5.1. Joint Distribution Functions.

5.2. Independent Random Variables.

5.3. Sums of Independent Random Variables. Identically Distributed Uniform Random Variables. Gamma Random Variables. Normal Random Variables. Poisson and Binomial Random Variables. Geometric Random Variables.

- 5.4. Conditional Distributions: Discrete Case.
- 5.5. Conditional Distributions: Continuous Case.
- 5.6. Joint Probability Distribution of Functions of Random Variables.

6. **Properties of Expectation (2 courses)**

- 6.1. Expectation of Sums of Random Variables.
- 6.2. Moments of the Number of Events that Occur.
- 6.3. Covariance, Variance of Sums, and Correlations.
- 6.4. Conditional Expectation. Computing Expectations by Conditioning.

Computing Probabilities by Conditioning . Conditional Variance.

- 6.5. Conditional Expectation and Prediction.
- 6.6. Moment Generating Functions.

7. Limit Theorems (1 course)

- 7.1. Markov's inequality.
- 7.2. Chebyshev's Inequality.
- 7.3. The Weak Law of Large Numbers.
- 7.4. The Central Limit Theorem.
- 7.5. The Strong Law of Large Numbers.

8. Mathematical statistics (3 courses)

8.1. Selection theory.

8.2. Estimation theory. Maximum likelihood method. Method of moments. Confidence interval method.

8.3. Statistics hypothesis testing. Z test. T test (Student). Tests for the comparison of average.

Bibliography :

1. Baron M., *Probability and statistics for computer scientists*, Taylor and Francis Group, LLC, 2007.

2. Grimmett G., Stirzaker D., *Probability and Random Processes*, Oxford University Press, 2001.

3. Grimmett G., Stirzaker D., *One Thousand Exercises in Probability*, Oxford University Press, 2001.

4. Ross S., *A first course in probability*, Pearson Education Inc., NJ, 2010. **Working language** : Romanian

Evaluation : Exam (partial exam)

Evaluation mode : Written examination